Book 10) Trigonometry

Topics : Right Triangle Trigonometry,
Trigonometric Equations,
Properties of Triangles &
Inverse Trigonometric Functions

Topic i) Right Triangle Trigonometry.

1. A pole stands vertically inside a triangular park *ABC*. If the angle of elevation of the top of the pole from each corner of the park is the same, then in ΔABC, the foot of the pole is at the

(a) centroid (b) circumcentre
(c) incentre (d) orthocentre

2. The length of the shadows of a vertical pole of height *h*, thrown by the sun's rays at three different moments are *h*, 2*h* and 3*h*. The sum of the angles of elevation of the rays at these three moments is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

3. Three vertical poles of heights h_1, h_2 and h_3 at the vertices *A*, *B* and *C* of ΔABC subtend angles α, β and γ respectively at the circumcentre of the triangle. If $\cot\alpha, \cot\beta$ and $\cot\gamma$ are in A.P., then h_1, h_2, h_3 are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

4. A circular ring of radius 3 cm is suspended horizontally from a point 4 cm vertically above the centre by 4 strings attached at equal intervals to its circumference. If the angle between two consecutive strings be θ, then $\cos\theta =$

(a) 4/5 (b) 4/25
(c) 16/25 (d) none of these

5. A tower stands at the centre of a circular park. *A* and *B* are two points on the boundary of the park such that *AB* (= *a*) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from *A* or *B* is 30°. The height of the tower is

(a) $\frac{a}{\sqrt{3}}$ (b) $\sqrt{3}a$
(c) $\frac{2a}{\sqrt{3}}$ (d) $2\sqrt{3}a$

6. The angle of elevation of a ladder leaning against a wall is

(a) 16.12 m (b) 17.26 m
(c) 18.11 m (d) none of these

7. A spherical balloon whose radius is *r* cm subtends at an observer's eye an angle α, when an angular deviation of the centre is β. The height of the centre of the balloon is

(a) $r\,\text{cosec}\frac{\alpha}{2}$ (b) $r\,\text{cosec}\frac{\alpha}{2}\sin\beta$
(c) $r\,\text{cosec}\alpha\ \text{cosec}\beta$ (d) none of these

8. The angles of elevation of the top of a T.V. tower from three points A, B, C in a straight line through the foot of the tower are α, 2α, 3α, respectively. If $AB = a$, the height of the tower is

(a) $a\tan\alpha$ (b) $a\sin\alpha$

(c) $a\sin 2\alpha$ (d) $a\sin 3\alpha$

9. From the bottom of a pole of height h, the angle of elevation of the top of a tower is α. The pole subtends an angle β at the top of a tower. The height of the tower is

(a) $\dfrac{h\sin\alpha\cos(\alpha+\beta)}{\cos\beta}$ (b) $\dfrac{h\sin\alpha\cos(\alpha-\beta)}{\sin\beta}$

(c) $\dfrac{h\sin\alpha\sin(\alpha+\beta)}{\sin\beta}$ (d) $\dfrac{h\sin\alpha\sin(\alpha-\beta)}{\sin\beta}$

10. ABC is a triangular park with $AB = AC = 100$ m. A clock tower is situated at the mid-point of BC. The angles of elevation of the top of the tower at A and B are $\cot^{-1}3.2$ and $\operatorname{cosec}^{-1}2.6$, respectively. The height of the tower is

(a) 16 m (b) 25 m

(c) 50 m (d) none of these

11. The angle of elevation of a cloud from a point h metres above is $\theta°$ and the angle of depression of its reflection in the lake is ϕ. Then the height is

(a) $\dfrac{h\sin(\phi-\theta)}{\sin(\phi+\theta)}$ (b) $\dfrac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$

(c) $\dfrac{h\sin(\theta+\phi)}{\sin(\theta-\phi)}$ (d) none of these

12. Each side of a square subtends an angle of 60° at a tower h metres high standing in the centre of the square. If a is the length of each side of the square, then

(a) $2a^2 = h^2$ (b) $2h^2 = a^2$

(c) $3a^2 = 2h^2$ (d) $2h^2 = 3a^2$

ANSWERS

1. (b)	**2.** (a)	**3.** (a)	**4.** (c)	**5.** (a)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (b)
11. (b)	**12.** (b)								

Topic ii) Trigonometric Equations

Also requires knowledge of graphs of trigonometric functions.

1. If $\sin A = \frac{336}{625}$, where $450° < A < 540°$, then $\sin\frac{A}{4} =$

(a) 3/5 (b) –(3/5)

(c) 4/5 (d) –(4/5)

2. If $x, y > 0$, then the range of

$\sin^{-1}\left(\frac{x}{1+x^2}\right) + \sin^{-1}\left(\frac{2y}{1+y^2}\right)$ is

(a) $[0, \pi]$ (b) $(-\pi, \pi]$

(c) $\left[0, \frac{2\pi}{3}\right]$ (d) $\left(0, \frac{2\pi}{3}\right]$

3. If $\cos A + \cos B = m$ and $\sin A + \sin B = n$, where m, $n \neq 0$, then $\sin(A+B)$ is equal to

(a) $\frac{mn}{m^2+n^2}$ (b) $\frac{2mn}{m^2+n^2}$

(c) $\frac{m^2+n^2}{2mn}$ (d) $\frac{mn}{m+n}$

4. If $y = \dfrac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta}$, then

(a) $\frac{1}{3} < y < 3$ (b) $y \notin \left[\frac{1}{3}, 3\right]$

(c) $-3 < y < -\frac{1}{3}$ (d) none of these

5. The number of solutions of the equation $\tan^{-1}\left(\dfrac{x}{1-x^2}\right) + \tan^{-1}\left(\dfrac{1}{x^3}\right) = \dfrac{3\pi}{4}$, belonging to the interval (0, 1), is

(a) 0 (b) 1 (c) 2 (d) infinite

6. The number of values of x between 0 and 2π that satisfy the equation $\sin x + \sin 2x + \sin 3x = \cos x + \cos 2x$ is

(a) 3 (b) 4 (c) 5 (d) 6

7. $\cos(540° - \theta) - \sin(630° - \theta)$ is equal to

(a) 0 (b) 2 cos θ

(c) 2 sin θ (d) $\sin\theta - \cos\theta$

8. The number of solutions of $\dfrac{\tan 3x - \tan 2x}{1 + \tan 3x \tan 2x} = 1$ in $[-2\pi, 2\pi]$ is

(a) 2 (b) 4

(c) 8 (d) none of these

9. $2\sec^2\alpha - \sec^4\alpha - 2\text{cosec}^2\alpha + \text{cosec}^4\alpha = 15/4$ if $\tan\alpha$ is equal to

(a) $1/\sqrt{2}$ (b) $1/2$ (c) $1/2\sqrt{2}$ (d) $1/4$

10. If $\cos p\theta + \cos q\theta = 0$, then the different values of θ are in A.P. whose common difference is

(a) $\dfrac{\pi}{p+q}$ (b) $\dfrac{\pi}{p-q}$ (c) $\dfrac{2\pi}{p \pm q}$ (d) $\dfrac{3\pi}{p \pm q}$

11. If $\dfrac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha} = x$, then $\dfrac{\cos\alpha}{1 + \sin\alpha}$ is equal to

(a) $1/x$ (b) x (c) $1 + x$ (d) $1 - x$

12. The number of solutions of the equation $(2 + \sqrt{3})\cos x = 1 - \sin x$ in the interval $[-\pi, \pi]$ is

(a) 2 (b) 3 (c) 4 (d) none of these

13. If tan 25° = x, then $\dfrac{\tan 155° - \tan 115°}{1 + \tan 155° \tan 115°}$ is equal to

(a) $\dfrac{1-x^2}{2x}$ (b) $\dfrac{1+x^2}{2x}$

(c) $\dfrac{1+x^2}{1-x^2}$ (d) $\dfrac{1-x^2}{1+x^2}$

14. The number of soultions of the equation $\text{cosec}\,\theta - \cot\theta = 1$ in $[0, 2\pi]$ is

(a) 4 (b) 3 (c) 2 (d) 1

15. If A = 130° and $x = \sin A + \cos A$, then

(a) $x > 0$ (b) $x < 0$ (c) $x = 0$ (d) $x \geq 0$

16. The number of all possible triplets (a_1, a_2, a_3) such that $a_1 + a_2\cos 2x + a_3\sin^2 x = 0$ for all x is

(a) 0 (b) 1 (c) 3 (d) infinite

17. If $\dfrac{\sin x}{a} = \dfrac{\cos x}{b} = \dfrac{\tan x}{c} = k$, then $bc + \dfrac{1}{ck} + \dfrac{ak}{1+bk}$ is

(a) $k\left(a + \dfrac{1}{a}\right)$ (b) $\dfrac{1}{k}\left(a + \dfrac{1}{a}\right)$

(c) $\dfrac{1}{k^2}$ (d) $\dfrac{a}{k}$

18. The general value of θ satisfying both the equations $\cos\theta = \dfrac{-1}{\sqrt{2}}$ and tan $\theta = 1$ is

(a) $n\pi + (\pi/4)$ (b) $n\pi + (5\pi/4)$

(c) $2n\pi + (\pi/4)$ (d) $2n\pi + (5\pi/4)$

19. The minimum value of $27\tan^2\theta + 3\cot^2\theta$ is

(a) 9 (b) 18 (c) 27 (d) 30

ANSWERS

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (c)
11. (d)	**12.** (a)	**13.** (a)	**14.** (d)	**15.** (a)	**16.** (d)	**17.** (b)	**18.** (d)	**19.** (b)	

Topic iii) Properties of Triangle

1. If the base angles of a triangle are $22\frac{1}{2}^\circ$ and $112\frac{1}{2}^\circ$, then the altitude of the triangle is equal to

(a) the base (b) $\frac{1}{3}$rd of the base

(c) $\frac{1}{2}$ of the base (d) $\frac{1}{4}$th of the base

2. If the sides *a*, *b*, *c* of a triangle *ABC* are the roots of $x^3-11x^2+38x-40=0$, then $\sum\frac{\cos A}{a}=$

(a) 3/4 (b) 1
(c) 9/16 (d) none of these

3. If $x, y > 0$, then the triangle whose sides are given by $3x + 4y$, $4x + 3y$ and $5x + 5y$ units is
(a) right-angled (b) obtuse-angled
(c) acute-angled (d) none of these

4. If in a triangle ABC, $\frac{a\sin B+b\sin A}{\sqrt{\sin A\sin B}}=4$, $\angle C=\frac{\pi}{3}$, then $a^2+b^2-c^2=$
(a) 4 (b) 6
(c) 8 (d) 10

5. If $A=60°$, $\frac{b}{c+a}+\frac{c}{a+b}=$
(a) 0 (b) 1
(c) s (d) S

6. In a triangle, $\cot A:\cot B:\cot C=30:19:6$, then $a:b:c$
(a) 5 : 6 : 7 (b) 6 : 7 : 5
(c) 7 : 6 : 5 (d) none of these

7. If $(a-b)(s-c)=(b-c)(s-a)$, then r_1, r_2, r_3 are in
(a) H.P. (b) G.P.
(c) A.P. (d) none of these

8. If twice the square of the diameter of a circle is equal to the sum of the squares of the sides of the inscribed triangle ABC, then $\sin^2A+\sin^2B+\sin^2C$ is equal to
(a) 2 (b) 3
(c) 4 (d) 1

9. In a triangle ABC, if $\frac{a}{1}=\frac{b}{\sqrt{3}}=\frac{c}{2}$, then
(a) $A+B-C=90°$
(b) the triangle is acute-angled
(c) A, B, C are in A.P.
(d) the triangle is abtuse-angled

10. The expression $\frac{(a+b+c)(b+c-a)(c+a-b)(a+b-c)}{4b^2c^2}$ is equal to
(a) $\cos^2 A$ (b) $\sin^2 A$
(c) $\cos A\cos B\cos C$ (d) none of these

11. In a triangle $ABC, (a+b+c)(b+c-a)=\lambda\, bc$ if
(a) $\lambda > d$ (b) $\lambda > 0$
(c) $0<\lambda<4$ (d) $\lambda > 4$

12. In a triangle ABC, $2ac\sin\frac{(A-B+C)}{2}$ is equal to
(a) $a^2+b^2-c^2$ (b) $c^2-a^2-b^2$
(c) $a^2-a^2-a^2$ (d) $c^2+a^2-b^2$

13. In any triangle ABC, $b^2\sin 2C+c^2\sin 2C+c^2\sin 2B$ is equal to
(a) Δ (b) 2Δ
(c) 3Δ (d) 4Δ

14. If a, b, A are given in a triangle and c_1 and c_2 are two possible values of the third side such that $c_1^2+c_1c_2+c_2^2=a^2$, then A is equal to
(a) 30° (b) 60°
(c) 90° (d) 145°

15. In a triangle ABC, if $s-a$, $s-b$, $s-c$ are in G.P., then $\frac{\sin^2A+\sin^2C}{\sin A+\sin C}=$
(a) $\sin B$ (b) $\cos B$
(c) $\sin[(a+C)/2]$ (d) $\sin[(A-C)/2]$

16. If $\cos A=\frac{\sin B}{2\sin C}$, then ΔABC is
(a) equilateral (b) isosceles
(c) right-angled (d) none of these

17. If in a triangle ABC, $\frac{\cos A}{a}=\frac{\tan C}{c}$, then $\sin(B+C)$ is equal to
(a) $\cos B\cos C$ (b) $\cos A\cos C$
(c) $\cos A\cos B$ (d) $\sin B\sin C$

18. In a triangle ABC, $1-\tan(A/2)\tan(B/2)$ is equal to
(a) $\frac{2a}{b+c-a}$ (b) $\frac{2b}{c+a-b}$
(c) $\frac{2c}{a+b-c}$ (d) $\frac{2c}{a+b+c}$

19. In a triangle ABC, if $\tan(A/2) = p$ and $\tan(B/2) = q$, then $\frac{2(p+q)(1-pq)}{(1+p^2)(1+q^2)}$ is equal to

(a) $\sin A$ (b) $\sin B$
(c) $\sin C$ (d) $\sin A + \sin B$

20. If R is the circumradius of a triangle ABC, then the area of its pedal triangle is

(a) $(1/2)R^2 \sin A \sin B \sin C$
(b) $(1/2)R^2 \sin 2A \sin 2B \sin 2C$
(c) $(1/2)R^2 \cos A \cos B \cos C$
(d) $(1/2)R^2 \cos 2A \cos 2B \cos 2C$

ANSWERS

1. (c)	**2.** (c)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (c)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (b)	**15.** (a)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (c)	**20.** (b)

Topic (V) Inverse Trigonometric Functions

1. $\cos^{-1}\left(\frac{1}{2}x^2+\sqrt{1-x^2}\cdot\sqrt{1-\frac{x^2}{4}}\right)=\cos^{-1}\frac{x}{2}-\cos^{-1}x$ holds for
   (a) $|x|\le 1$ (b) $x\in\mathbb{R}$
   (c) $0\le x\le 1$ (d) $-1\le x\le 0$

2. A solution to the equation $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}$ is
   (a) $x=1$ (b) $x=-1$
   (c) $x=0$ (d) $x=\pi$

3. The set of values of a for which $x^2+ax+\sin^{-1}(x^2-4x+5)+\cos^{-1}(x^2-4x+5)=0$ has at least one solution is
   (a) $(-\infty,-\sqrt{2\pi}]\cup[\sqrt{2\pi},\infty)]$
   (b) $(-\infty,-\sqrt{2\pi})\cup(\sqrt{2\pi},\infty)$
   (c) $\mathbb{R}$
   (d) none of these

4. If $\cos^{-1}x>\sin^{-1}x$, then
   (a) $x<0$ (b) $-1<x<10$
   (c) $0\le x<\frac{1}{\sqrt{2}}$ (d) $-1\le x<\frac{1}{\sqrt{2}}$

5. All possible values of p and q for which
   $$\cos^{-1}\sqrt{p}+\cos^{-1}\sqrt{1-p}+\cos^{-1}\sqrt{1-q}=\frac{3\pi}{4}$$
   holds are
   (a) $p=1,\ q=\frac{1}{2}$ (b) $q>1,\ p=\frac{1}{2}$
   (c) $0\le p\le 1,\ q=\frac{1}{2}$ (d) none of these

6. Which one of the following is correct?
   (a) $\tan 1>\tan^{-1}1$ (b) $\tan 1<\tan^{-1}1$
   (c) $\tan 1=\tan^{-1}1$ (d) none of these

7. The number of solutions of the equation $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$ is
   (a) 0 (b) 1
   (c) 2 (d) more than two

8. The value of $\cos\left(\frac{1}{2}\cos^{-1}\frac{1}{8}\right)$ is equal to
   (a) $\frac{3}{4}$ (b) $-\frac{3}{4}$
   (c) $\frac{1}{16}$ (d) $\frac{1}{4}$

9. The value of $\tan^{-1}\left(\frac{a}{b+c}\right)+\tan^{-1}\left(\frac{b}{c+a}\right)$, if $\angle C=90°$ in a triangle ABC, is
   (a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
   (c) $\frac{\pi}{2}$ (d) π

10. The number of solutions of the equation $\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is
   (a) 0 (b) 1
   (c) 2 (d) 3

11. If $u=\cot^{-1}\sqrt{\tan\alpha}-\tan^{-1}\sqrt{\tan\alpha}$, then $\tan\left(\frac{\pi}{4}-\frac{u}{2}\right)$ is equal to

(a) $\sqrt{\tan\alpha}$ (b) $\sqrt{\cot\alpha}$
(c) $\tan\alpha$ (d) $\cot\alpha$

12. The value of $\cot^{-1}3+\operatorname{cosec}^{-1}\sqrt{5}$ is
(a) $\pi/3$ (b) $\pi/2$
(c) $\pi/4$ (d) $\pi/6$

13. If $2\sin^{-1}x-3\cos^{-1}x=4$, then $2\sin^{-1}x+3\cos^{-1}x$ equals

(a) $\frac{6\pi-4}{5}$ (b) $\frac{4-6\pi}{5}$
(c) $3\pi/2$ (d) 0

14. $\sum_{r=1}^{\infty}\cot^{-1}\left(r^2+\frac{3}{4}\right)$ equals

(a) $\frac{\pi}{2}$ (b) $\cot^{-1}2$
(c) $\frac{\pi}{6}$ (d) $\tan^{-1}2$

15. Given $\pi<\theta<\frac{3\pi}{2}$, then the value of the expression

$\sqrt{4\sin^4\theta+\sin^2 2\theta}+4\cos^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ is

(a) 2 (b) $2+4\sin\theta$
(c) $2-4\sin\theta$ (d) none of these

ANSWERS

1. (c)	**2.** (c)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (a)	**10.** (b)
11. (a)	**12.** (c)	**13.** (a)	**14.** (c)	**15.** (a)					