Book 2) Complex Numbers

Topics Covered: Quadratic Equations & Complex Numbers.

1. The equation $(a^2 - a - 2)x^2 + (a^2 - 4)x + a^2 - 3a + 2 = 0$ will have more than two solutions if a equals
(a) 2 (b) 1
(c) –2 (d) not possible

2. If the difference between the roots of the equation $x^2 + px + q = 0$ is 1, then
(a) $p^2 = 4q$ (b) $p^2 = 4q + 1$
(c) $p^2 = 4q - 1$ (d) none of these

3. If $0 < a < b < c$ and the roots α and β of the equation $ax^2 + bx + c = 0$ are imaginary, then
(a) $|\alpha| \neq |\beta|$ (b) $|\alpha| > 1$
(c) $|\beta| < 1$ (d) none of these

4. Ramesh and Mahesh solve a quadratic equation. Ramesh reads its constant term wrongly and finds its roots as 8 and 2 whereas Mahesh reads the coefficient of x wrongly and finds its roots as 11 and –1. The correct roots of the equation are
(a) 11, 1 (b) –11, 1
(c) 11, –1 (d) none of these

5. If the roots of $ax^2 + bx + c = 0$ are α and β and the roots of $Ax^2 + Bx + C = 0$ are $\alpha - K$ and $\beta - K$, then $(B^2 - 4AC)/(b^2 - 4ac)$ is equal to
(a) 0 (b) 1 (c) $(A/a)^2$ (d) $(a/A)^2$

6. If α and β are the roots of $x^2 + px + q = 0$ and α^4 and β^4 are the roots of $x^2 - rx + 5 = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always
(a) two real roots
(b) two negative roots
(c) two positive roots
(d) one positive and one negative root

7. If $1, \omega, \omega^2, \ldots, \omega^{n-1}$ are the nth roots of unity, then $(1-\omega)(1-\omega^2)\ldots(1-\omega^{n-1})$ equals
(a) 0 (b) 1 (c) n (d) n^2

8. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, then it must be equal to
(a) $\frac{pq' - p'q}{q - q'}$ (b) $\frac{p'q - pq'}{q - q'}$
(c) $\frac{q + q'}{p' - p}$ (d) none of these

9. If the equation $\frac{x^2 - bx}{ax - c} = \frac{m-1}{m+1}$ has roots equal in magnitude but opposite in sign, then $m = ?$
(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$
(c) $\frac{b-c}{b+a}$ (d) none of these

10. If one root of the equations $x^2 + px + q = 0$ and $x^2 + p'x + q^2 = 0$ ($p \neq p'$ and $q \neq q'$) is common, then the root is
(a) $\frac{q - q'}{p - p'}$ (b) $\frac{pq' - p'q}{q - q'}$
(c) $\frac{q - q'}{p' - p}$ or $\frac{pq' - p'q}{q - q'}$ (d) $\frac{q - q'}{p' - p}$ or $\frac{pq - p'q'}{q - q'}$

11. If one root of the equation $x^2 - 30x + p = 0$ is the square of the other, then p is equal to
(a) only 125 (b) 125, –216
(c) 125, 215 (d) only 216

12. The value of $\sqrt{8 + 2\sqrt{8 + 2\sqrt{8 + 2\sqrt{8 + \cdots}}}}$
(a) 10 (b) 6
(c) 8 (d) 4

13. If $\tan\theta$ and $\sec\theta$ are the roots of the equation $ax^2 + bx + c = 0$, then
(a) $a^3 + b^3 + c^3 - 3abc = 0$ (b) $a^2 + b^2 + 2ac = 0$
(c) $a^4 + 4ab^2c - b^4 = 0$ (d) none of these

14. Let $\alpha + \beta$, $\alpha, \beta \in \mathbb{R}$, $\beta \neq 0$ be a root of the equation $x^3 + ax + b = 0$, where $a, b \in \mathbb{R}$. Then the cubic equation with real coefficients, one of whose roots is α, is
(a) $x^3 - ax + b = 0$ (b) $x^3 - 2ax + b = 0$
(c) $8x^3 + 2ax - b = 0$ (d) $8x^3 + 2ax + b = 0$

15. If the roots of the equation $bx^2 + cx + a = 0$ are imaginary, then for all real values of x the expression $3b^2x^2 + 6bcx + 2c^2$ is
(a) greater than $4ab$ (b) less than $4ab$
(c) greater than $-4ab$ (d) less than $-4ab$

16. If $a \in Z$ and the equation $(x - a)(x - 10) + 1 = 0$ has integral roots, then the values of a are
(a) 10, 8 (b) 12, 10
(c) 12, 8 (d) none of these

17. The values of the parameter a for which the quadratic equations $(1 - 2a)x^2 - 6ax - 1 = 0$ and $ax^2 - x + 1 = 0$ have at least one root in common are
(a) $0, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{2}{9}$
(c) $\frac{2}{9}$ (d) $0, \frac{1}{2}, \frac{2}{9}$

18. Let $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$, $\alpha = a + a^2 + a^4$ and $\beta = a^3 + a^5 + a^6$. Then the equation whose roots are α and β is
(a) $x^2 - x + 2 = 0$ (b) $x^2 + x - 2 = 0$
(c) $x^2 - x - 2 = 0$ (d) $x^2 + x + 2 = 0$

ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (c)	**8.** (a)	**9.** (b)	**10.** (c)
11. (a)	**12.** (d)	**13.** (c)	**14.** (a)	**15.** (c)	**16.** (c)	**17.** (c)	**18.** (d)		

Topic ii) Complex Numbers

1. If $z+\sqrt{2}|z+1|+i=0$ and $z=x+iy$, then
(a) $x=-2$ (b) $x=2$
(c) $y=-2$ (d) $y=1$

2. If arg $(z) \le 0$, then arg $(-z) - \arg(z)$ is equal to
(a) -1 (b) π
(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

3. If α is the nth root of unity, then $1 + 2\alpha + 3\alpha^2 + \cdots$ to n terms is equal to
(a) $\frac{-n}{(1-\alpha)^2}$ (b) $\frac{-n}{1-\alpha}$
(c) $\frac{-2n}{1-\alpha}$ (d) $\frac{-2n}{(1-\alpha)^2}$

4. In G.P., if the first term and the common ratio are both $\frac{1}{2}(\sqrt{3}+i)$, then the absolute value of its nth term is
(a) 1 (b) 2^n
(c) 4^n (d) none of these

5. Find the least value of n ($n \in N$) for which $\left(\frac{1+i}{1-i}\right)^n$ is real.
(a) 1 (b) 2
(c) 3 (d) 4

6. If $(1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \cdots + a_{2n}x^{2n}$, then $a_0 + a_3 + a_6 + a_9 + \cdots$ is equal to
(a) 3^n (b) 3^{n-1}
(c) 3^{n+1} (d) none of these

7. If $|z| = \max\left[|z-1|, |z+1|\right]$, then
(a) $|z+\bar{z}| = \frac{1}{2}$ (b) $z+\bar{z}=1$
(c) $|z+\bar{z}|=1$ (d) none of these

8. If $z_1 = -3+5i$, $z_2 = -5-3i$ and z is a complex number lying on the line segment joining z_1 and z_2, then arg(z) can be
(a) $-\frac{3\pi}{4}$ (b) $-\frac{\pi}{4}$
(c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{6}$

9. The expression $\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^n - \frac{1+i\tan n\alpha}{1-i\tan n\alpha}$ when simplified reduces to
(a) zero (b) $2\sin n\alpha$
(c) $2\cos n\alpha$ (d) none of these

10. If $z = x+iy$ satisfies $\arg(z-1) = \arg(z+3i)$, then the value of $(x-1):y$ is equal to
(a) 2 : 1 (b) 1 : 3
(c) –1 : 3 (d) none of these

11. The points z_1, z_2, z_3, z_4 in the complex plane taken in order are the vertices of a parallelogram if and only if,
(a) $z_1+z_2=z_3+z_4$ (b) $z_1+z_3=z_2+z_4$
(c) $z_1z_2=z_3z_4$ (d) none of these

12. If P is the affix of z in the Argand diagram and P moves so that $(z-i)/(z+1)$ is always purily imaginary, then the locus of z is
(a) circle centre [(1/2), (1/2)], radius $1/\sqrt{2}$
(b) circle centre [–(1/2), –(1/2)], radius $1/\sqrt{2}$
(c) circle centre (2, 2) and radius (1/2)
(d) none of these

13. Let A, B, C represent the complex numbers z_1, z_2, z_3, respectively, on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number
(a) $z_1+z_2-z_3$ (b) $z_2+z_3-z_1$
(c) $z_3+z_1-z_2$ (d) $z_1+z_2+z_3$

14. If a, b, c are integers not all equal and ω is a cube root of unity ($\omega \neq 1$), then the minimum value of $|a + b\omega + c\omega^2|$ is

(a) 1 (b) 0

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

15. If z_1 and z_2 are two complex numbers and if $\arg\left(\frac{z_1 + z_2}{z_1 - z_2}\right) = \frac{\pi}{2}$ but $|z_1 + z_2| \neq |z_1 - z_2|$, then the figure formed by the points represented by 0, z_1, z_2 and $z_1 + z_2$ is

(a) a parallelogram but not a rectangle or a rhombus

(b) a rectangle but not a square

(c) a rhombus but not a square

(d) a square

16. If $\prod_{p=1}^{r} e^{ip\theta} = 1$, where Π denotes the continued product, then the most general value of θ is

(a) $\frac{2n\pi}{r(r-1)}$ (b) $\frac{2n\pi}{r(r+1)}$

(c) $\frac{4n\pi}{r(r-1)}$ (d) $\frac{4n\pi}{r(r+1)}$

17. $\sin^{-1}\left[\frac{1}{i}(z-1)\right]$, where z is non-real, can be the angle of a triangle if

(a) $\text{Re}(z) = 1, \text{Im}(z) = 2$

(b) $\text{Re}(z) = 1, 0 < \text{Im}(z) \leq 1$

(c) $\text{Re}(z) + \text{Im}(z) = 0$

(d) none of these

18. If $|z - i| \leq 2$ and $z_0 = 5 + 3i$, then the maximum value of $|iz + z_0|$ is

(a) $2 + \sqrt{31}$ (b) 7

(c) $\sqrt{31} - 2$ (d) none of these

19. Multiplying a complex number by i rotates the vector representing the complex number through an angle of

(a) 180° (b) 90°

(c) 60° (d) 360°

20. The area of the triangle whose vertices are represented by the complex numbers, 0, z, $ze^{i\alpha}$ $(0 < \alpha < \pi)$ is equal to

(a) $\frac{1}{2}|z|^2 \cos\alpha$ (b) $\frac{1}{2}|z|^2 \sin\alpha$

(c) $\frac{1}{2}|z|^2 \sin\alpha \cos\alpha$ (d) $\frac{1}{2}|z|^2$

ANSWERS

1. (a)	**2.** (b)	**3.** (b)	**4.** (a)	**5.** (b)	**6.** (b)	**7.** (d)	**8.** (d)	**9.** (a)	**10.** (b)
11. (b)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (c)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (b)	**20.** (b)