Book 3) Counting Principles, PnC, Probability & statistics

Topic i) Counting Principles & PnC.

1. A train going from London to Cambridge stops at 12 intermediate stations. Seventy-five persons enter the train during the journey with 75 different tickets of the same class. The number of different sets of tickets they may be holding is

(a) $^{78}C_3$ (b) $^{91}C_{75}$
(c) $^{84}C_{75}$ (d) none of these

2. Seven different coins are to be divided amongst three persons. If no two of the persons receive the same number of coins but each receives at least one coin and none is left over, then the number of ways in which the division may be made is

(a) 420 (b) 630
(c) 710 (d) none of these

3. The streets of a city are arranged like the lines of a chess board. There are m streets running north to south and n streets running east to west. The number of ways in which a man can travel from NW to SE corner going the shortest possible distance is

(a) $\sqrt{m^2+n^2}$ (b) $\sqrt{(m-1)^2\cdot(n-1)^2}$
(c) $\dfrac{(m+n)!}{m!\cdot n!}$ (d) $\dfrac{(m+n-2)!}{(m-1)!\cdot(n-1)!}$

4. The number of ordered triplets of positive integers which are solutions of the equation $x + y + z = 100$ is

(a) 5081 (b) 6005
(c) 4851 (d) none of these

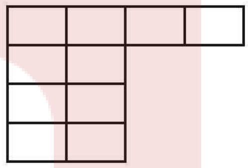
5. The number of ways in which a batsman can score 14 runs in 6 balls not scoring more than 4 runs in any ball is

(a) $^{19}C_5-6\cdot{}^{14}C_5+15\cdot{}^{9}C_5$
(b) $^{19}C_5+6\cdot{}^{14}C_5-15\cdot{}^{9}C_5$
(c) 386
(d) 286

6. There are n numbered seats around a round table. The number of ways in which $m(m < n)$ persons can sit around the round table is

(a) $^{n}C_m\cdot m!$ (b) $^{n}C_m-(m-1)!$
(c) $\dfrac{^{n}C_m-(m-1)!}{2}$ (d) none of these

7. The number of ways in which 5X's cab be placed in the squares of Fig. so that no row remains empty is



(a) 97 (b) 44
(c) 100 (d) 126

8. Two variants of a test paper are distributed among 12 students. The number of ways of seating of the students in two rows so that the students sitting side by side do not have identical papers and those sitting in the same column have the same paper is

(a) $\dfrac{12!}{6!6!}$ (b) $\dfrac{(12)!}{2^5\cdot 6!}$
(c) $(6!)^2\cdot 2$ (d) $12!\times 2$

9. Let P_n denotes the number of ways of selecting 3 people out of n sitting in a row if no two of them are consecutive and Q_n is the corresponding figure when they are in a circle. If $P_n - Q_n = 6$, then n is equal to

(a) 8 (b) 9
(c) 10 (d) 12

10. The number of times the digits 3 will be written when listing the integers from 1 to 1000 is

(a) 269 (b) 300
(c) 271 (d) 302

11. If $x \in N$ and $^{x-1}C_4 - {}^{x-1}C_3 - \frac{5}{4}\,{}^{x-2}P_2 < 0$, then x is equal to

(a) 4, 5, 6, 7, 8, 9, 10 (b) $3 < x < 11$
(c) $3 \le x \le 10$ (d) none of these

12. The number of non-negative integral solutions of the equation $x + y + 3z = 33$ is

(a) 120 (b) 135
(c) 210 (d) 520

13. A is a set containing n different elements. A subset P of A is chosen. The set A is reconstructed by replacing the elements of P. A subset Q of A is again chosen. The number of ways of choosing P and Q so that $P \cap Q$ contains exactly two elements is

(a) ${}^nC_3 \cdot 2^n$ (b) ${}^nC_2 \cdot 3^{n-2}$
(c) 3^{n-2} (d) none of these

14. The number of ways in which 2 Indians, 3 Americans, 3 Italians and 4 French can be seated on a circle, if the people of the same nationality sit together is

(a) $2 \cdot (4!)^2\,(3!)^2$ (b) $2 \cdot (3!)^3 \cdot 4!$
(c) $2 \cdot (3!)\,(4!)^3$ (d) none of these

15. The number of ways in which 3 numbers in A.P. can be selected from 1, 2, 3, …, n is

(a) $\left(\frac{n-1}{2}\right)^2$ if n is even (b) $\frac{n(n-2)}{4}$ if n is odd
(c) $\frac{(n-2)^2}{4}$ if n is odd (d) $\frac{n(n-3)}{4}$ if n is even

16. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then the value of $C_1 + 2C_2 + 3C_3 + \cdots + nC_n$ equals

(a) $n2^n$ (b) $(n+1)2^n$
(c) $n2^{n-1}$ (d) $n \cdot 2^{n+1}$

17. If n is even and ${}^nC_0 < {}^nC_1 < {}^nC_2 < \ldots < {}^nC_r > {}^nC_{r+1} > {}^nC_{r+2} > \cdots > {}^nC_n$, then $r =$

(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$
(c) $\frac{n-2}{2}$ (d) $\frac{n+2}{2}$

18. If ${}^{13}C_r$ is denoted by C_r, then the value of $C_1 + C_5 + C_7 + C_9 + C_{11}$ is equal to

(a) $2^{12} - 287$ (b) $2^{12} - 165$
(c) $2^{12} - C_3$ (d) $2^{12} - C_2 - C_{13}$

ANSWERS

1. (a)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (a)	**7.** (b)	**8.** (d)	**9.** (c)	**10.** (b)
11. (a)	**12.** (c)	**13.** (b)	**14.** (b)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (a)		

Topic ii) Probability

1. A bag contains 7 tickets marked with the numbers 0, 1, 2, 3, 4, 5, 6, respectively. A ticket is drawn and replaced. Then the chance that after 4 drawings, the sum of the numbers drawn is 8 is
(a) 165/2401 (b) 149/2401
(c) 3/49 (d) none of these

2. A biased coin with probability p, $0 < p < 1$, of heads is tossed until a head appears for the first time. If the probability that the number of tosses required is even is 2/5, then p equals
(a) 1/3 (b) 2/3
(c) 2/5 (d) 3/5

3. Numbers 1, 2, 3, …, 100 are written down on each of the cards A, B and C. One number is selected at random from each of the cards. The probability that the numbers so selected can be the measures (in cm) of three sides of a right-angled triangle, no two of which are similar, is

(a) $\frac{4}{100^3}$ (b) $\frac{3}{50^3}$

(c) $\frac{3!}{100^3}$ (d) none of these

4. Three numbers are chosen at random without replacement from the set $A = \{x | 1 \le x \le 10, x \in N\}$. The probability that the minimum of the chosen numbers is 3 and maximum is 7 is

(a) 1/12 (b) 1/15

(c) 1/40 (d) none of these

5. In a purse, there are 10 coins, all 5 naya paisa except one which is a rupee. In another purse, there are 10 coins all 5 naya paisa. Nine coins are taken out from the former purse and put into the latter and then 9 coins are taken out from the latter and put into the former. Then the chance that the rupee is still in the first purse is

(a) 9/19 (b) 10/19

(c) 4/9 (d) none of these

6. A letter is known to have come from either KRISHNAGIRI or DHARMAPURI. On the post mark only the two consecutive letters "RI" are visible. Then the chance that it came from KRISHNAGIRI is

(a) 3/5 (b) 2/3

(c) 9/14 (d) none of these

7. If $\frac{(1+3p)}{3}, \frac{(1-p)}{4}$ and $\frac{(1-2p)}{2}$ are the probabilities of three mutually exclusive events, then the set of all values of p is

(a) $\left[\frac{1}{2}, \frac{2}{3}\right]$ (b) $\left[\frac{1}{3}, \frac{1}{2}\right]$

(c) $\left[\frac{1}{4}, \frac{1}{2}\right]$ (d) $\left[\frac{1}{3}, \frac{2}{3}\right]$

8. Three natural numbers are taken at random from the set $A = \{x : 1 \le x \le 100, x \in N\}$. The probability that the A.M. of the numbers taken is 25 is

(a) $\frac{^{77}C_2}{^{100}C_3}$ (b) $\frac{^{25}C_2}{^{100}C_3}$

(c) $\frac{^{74}C_{72}}{^{100}C_{97}}$ (d) none of these

9. From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is

(a) 1/2 (b) 1/3

(c) 2/3 (d) none of these

10. The probability that out of 10 persons, all born in April, at least two have the same birthday is

(a) $\frac{^{30}C_{10}}{(30)^{10}}$ (b) $1 - \frac{^{30}C_{10}}{30!}$

(c) $\frac{(30)^{10} - {^{30}C_{10}}}{(30)^{10}}$ (d) none of these

11. Let p be the probability that a man aged x years will die in a year time. The probability that out of n men $A_1, A_2, A_3, \ldots, A_n$ each aged x years, A_1 will die and will be the first to die is

(a) $\frac{1-p^n}{n}$ (b) $\frac{p}{n}$

(c) $\frac{p(1-p)^{n-1}}{n}$ (d) $\frac{1-(1-p)^n}{n}$

12. The contents of urns I and II are as follows:

Urn I: 4 white and 5 black balls

Urn II: 3 white and 6 black balls

One urn is chosen at random and a ball is drawn and its colour is noted and replaced back to the urn. Again a ball is drawn from the same urn, colour is noted and replaced. The process is repeated 4 times and as a result, one ball of white colour and 3 of black colour are noted. Find the probability that the chosen urn was I.

(a) $\frac{125}{287}$ (b) $\frac{64}{127}$

(c) $\frac{25}{287}$ (d) $\frac{79}{192}$

13. A die is weighted so that the probability of different faces to turn up is as given

Number	1	2	3	4	5	6
Probability	0.2	0.1	0.1	0.3	0.1	0.2

If $P\left(\frac{A}{B}\right) = P_1$ and $P\left(\frac{B}{A}\right) = P_2$ and $P\left(\frac{C}{A}\right) = P_3$, then the values of P_1, P_2, P_3 respectively are [Take the events A, B and C as $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$ and $C = \{2, 4, 6\}$]

(a) $\frac{2}{3}, \frac{1}{3}, \frac{1}{4}$ (b) $\frac{1}{3}, \frac{1}{3}, \frac{1}{6}$

(c) $\frac{1}{4}, \frac{1}{3}, \frac{1}{6}$ (d) $\frac{2}{3}, \frac{1}{6}, \frac{1}{4}$

14. If one ball is drawn at random from each of the three boxes containing 3 white and 1 black, 2 white and 2 black, and 1 white and 3 black balls, then the probability that 2 white and 1 black balls be drawn is

(a) $\frac{13}{32}$ (b) $\frac{1}{4}$

(c) $\frac{1}{32}$ (d) $\frac{3}{16}$

15. A and B draw two cards each, one after another, from a well-shuffled pack of 52 cards. The probability that all the four cards drawn are the same suit is

(a) $\frac{44}{85\times 49}$ (b) $\frac{11}{85\times 49}$

(c) $\frac{13\times 24}{17\times 25\times 49}$ (d) none of these

16. Fifteen coupons are numbered 1, 2, 3, …, 15 respectively. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on a selected coupon is 9 is

(a) $\left(\frac{9}{16}\right)^6$ (b) $\left(\frac{8}{15}\right)^7$

(c) $\left(\frac{3}{5}\right)^7$ (d) $\frac{9^7-8^7}{15^7}$

17. Let A and B be two independent events such that $P(A)=\frac{1}{5}, P(A\cup B)=\frac{7}{10}$. Then $P(\bar{B})$ is equal to

(a) 3/8 (b) 2/7

(c) 7/9 (d) none of these

18. The probability that at least one of the events A and B occurs is 3/5. If A and B occur simultaneously with probability 1/5, then $P(A') + P(B')$ is

(a) 2/5 (b) 4/5

(c) 6/5 (d) 7/5

19. A coin is tossed 7 times. Each time a man calls a head, the probability that he wins the toss on more occasions is

(a) 1/4 (b) 5/8

(c) 1/2 (d) none of these

ANSWERS

1. (b)	**2.** (a)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (c)
11. (d)	**12.** (a)	**13.** (d)	**14.** (a)	**15.** (a)	**16.** (d)	**17.** (a)	**18.** (c)	**19.** (c)	

Topic iii) statistics

1. Which of the following is not a measure of central tendency?
(a) mean (b) median
(c) mode (d) range

2. The weighted mean of the first n natural numbers whose weights are equal to the squares of the corresponding numbers is
(a) $\frac{n+1}{2}$ (b) $\frac{3n(n+1)}{2(2n+1)}$
(c) $\frac{(n+1)(2n+1)}{6}$ (d) $\frac{n(n+1)}{2}$

3. The mean of the following frequency table is 50.

Class	Frequency
0–20	17
20–40	f_1
40–60	32
60–80	f_2
80–100	19
Total	120

The missing frequencies are
(a) 28, 24 (b) 24, 36
(c) 38, 28 (d) none of these

4. The geometric mean of 1, 2, 2^2, 2^3,…, 2^n is
(a) $2^{2/n}$ (b) $2^{n/2}$ (c) $2^{(n-1)/2}$ (d) $2^{(n+1)/2}$

5. If $\bar{x}$ is the mean of a distribution, then $\sum f_i(x_i - \bar{x}) =$
(a) 0 (b) M.D.
(c) S.D. (d) none of these

6. The variance of the first n natural numbers is
(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$
(c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

7. If $\bar{x}_1$ and $\bar{x}_2$ are means of two distributions such that $\bar{x}_1 < \bar{x}_2$ and $\bar{x}$ is the mean of the combined distribution, then
(a) $\bar{x} < \bar{x}_1$ (b) $\bar{x} > \bar{x}_2$
(c) $\bar{x} = \frac{\bar{x}_1 + \bar{x}_2}{2}$ (d) $\bar{x}_1 < \bar{x} < \bar{x}_2$

8. The A.M. of n observations is $\bar{x}$. If the sum of $n - 5$ observations is a, then the mean of the remaining 5 observations is
(a) $\frac{n\bar{x}+a}{5}$ (b) $\frac{n\bar{x}-a}{5}$
(c) $n\bar{x}+a$ (d) none of these

9. A car completes the first half of its journey with a velocity v_1, and the rest half with velocity v_2. Then, the average velocity of the car for the whole journey is
(a) $\frac{v_1+v_2}{2}$ (b) $\sqrt{v_1 v_2}$
(c) $\frac{2v_1 v_2}{v_1+v_2}$ (d) none of these

10. Which one of the following measures is the most suitable one of central location for computing intelligence of students?
(a) mode (b) A.M.
(c) G.M. (d) median

ANSWERS

1. (d) **2.** (b) **3.** (a) **4.** (b) **5.** (a) **6.** (a) **7.** (d) **8.** (b) **9.** (c) **10.** (d)