Book 3) Differential Calculus.

Topics Covered: Limits, Continuity, Differentiation & Applications.

1. If $\lim_{x\to\infty}\left(1+\frac{\lambda}{x}+\frac{\mu}{x^2}\right)^{2x}=e^2$, then

(a) $\lambda=1,\ \mu=2$
(b) $\lambda=2,\ \mu=1$
(c) $\lambda=1,\ \mu=$ any real constant
(d) $\lambda=\mu=1$

2. If α is a repeated root of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{\sin(ax^2+bx+c)}{(x-\alpha)^2}$ is

(a) 0 (b) a
(c) b (d) c

3. The function $f(x)=[x]^2-[x^2]$ (where [.] denotes the greatest integer function) is discontinuous at

(a) all integers
(b) all integers except 0 and 1
(c) all integers except 1
(d) all integers except 0

4. If $y=(\sin x)^{\sin x}$, then $dy/dx=$

(a) $(\sin x)^{\sin x}\cos x\,(1+\log\sin x)$
(b) $(\sin x)^{\sin x}(1+\log\sin x)$
(c) $(\sin x)^{\sin x}(1-\log\sin x)$
(d) none of these

5. Let $f(x+y)=f(x)\cdot f(y)$ for all x and y. If $f(5)=2$ and $f'(0)=3$ then $f'(5)$ is equal to

(a) 5 (b) 6
(c) 0 (d) none of these

6. $\lim_{x\to0}\frac{e^{px}-e^{qx}}{x}=$

(a) $\ln p/q$ (b) $\ln pq$
(c) $p-q$ (d) none of these

7. $\lim_{x\to0}\frac{\sin^{-1}x-\tan^{-1}x}{x^3}$ is equal to

(a) 2 (b) 1
(c) –1 (d) 1/2

8. $\lim_{x\to0}\frac{x^4\left(\cot^4 x-\cot^2 x+1\right)}{\left(\tan^4 x-\tan^2 x+1\right)}$ is equal to

(a) 1 (b) 0
(c) 2 (d) none of these

9. $\lim_{x\to-1+}\frac{\sqrt{\pi}-\sqrt{\cos^{-1}x}}{\sqrt{x+1}}$ is equal to

(a) $1/\sqrt{\pi}$ (b) $1/\sqrt{(2\pi)}$
(c) 1 (d) 0

10. Let $f:\mathbb{R}^+\to\mathbb{R}$ be a continuous function satisfying $f\left(\frac{x}{y}\right)=f(x)-f(y)$ for all $x,y\in\mathbb{R}^+$. If $f'(1)=1$, then

(a) f' is bounded (b) $\lim_{x\to0}f\left(\frac{1}{x}\right)=0$
(c) $\lim_{x\to0}\frac{f(x)}{x}=1$ (d) $\lim_{x\to0}x\cdot f(x)=0$

11. $f(x)$ and $g(x)$ are continuous in $[a, b]$ differentiable in (a, b) with $f(a)=4$, $f(b)=10$, $g(a)=1$, $g(b)=3$. Then for $a<c<b$, $f'(c)=kg'(c)$, where k is equal to

(a) 1/2 (b) 2
(c) 3 (d) none of these

12. $\lim_{x\to0}\left(\frac{4^x+9^x}{2}\right)^{1/x}=$

(a) 2 (b) 6
(c) 16 (d) none of these

13. Let $h(x)=\min\{x, x^2\}, x\in\mathbb{R}$. Then $h(x)$ is

(a) differentiable everywhere
(b) non-differentiable at three values of x
(c) non-differentiable at two values of x
(d) none of these

14. $\lim_{n\to\infty}\frac{2\cdot3^n-3\cdot5^n}{3\cdot3^n+4\cdot5^n}=$

(a) 2/3 (b) –3/4
(c) 1 (d) none of these

15. If $f(x)=x, x\le1$ and $f(x)=x^2+bx+c\ (x>1)$ and $f'(x)$ exists finitely for all $x\in\mathbb{R}$, then

(a) $b=-1, c\in\mathbb{R}$ (b) $c=1, b\in\mathbb{R}$
(c) $b=1, c=-1$ (d) $b=-1, c=1$

16. $\lim_{x\to\infty}\frac{(2+x)^{40}(4+x)^5}{(2-x)^{45}}=$

(a) –1 (b) 1
(c) 16 (d) 32

17. If the function $f(x)=\begin{cases}Ax-B, & x\le1\\ 3x, & 1<x<2\\ Bx^2-A, & x\ge2\end{cases}$

is continuous at $x=1$ and discontinuous at $x=2$, then

(a) $A=3+B, B\ne3$
(b) $A=3+B, B=3$
(c) $A=3+B$
(d) none of these

18. $\lim_{x\to\infty} \frac{x^n}{e^x} = 0 \quad (n \in I)$ for

(a) no value of n (b) all values of n
(c) negative values of n (d) positive values of n

19. Let $f''(x)$ be continuous at $x = 0$ and $f''(0) = 4$. Then $\lim_{x\to 0} \frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is equal to

(a) 11 (b) 2
(c) 12 (d) none of these

20. $\lim_{x\to\infty} \frac{8}{x^8}\left[1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$ is equal to

(a) $\frac{1}{16}$ (b) $-\frac{1}{16}$
(c) $\frac{1}{32}$ (d) $-\frac{1}{32}$

ANSWERS

1. (c)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (d)
11. (c)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (d)	**16.** (a)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (c)

Topic ii) Differentiation

1. If $f(x + y) = f(x)\,f(y)$ for all $x, y \in \mathbb{R}$ and $f(4) = 3$, $f'(0) = 4$, then $f'(4) =$

(a) 12 (b) 4
(c) 3 (d) 6

2. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a^3(x^3-y^3)$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{x^2}{y^2}\sqrt{\dfrac{1-y^6}{1-x^6}}$ (b) $\dfrac{y^2}{x^2}\sqrt{\dfrac{1-y^6}{1-x^6}}$

(c) $\dfrac{x^2}{y^2}\sqrt{\dfrac{1-x^6}{1-y^6}}$ (d) none of these

3. If $y=\log(\sqrt{x-a}+\sqrt{x-b})$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{1}{2\sqrt{x-a}\sqrt{x-b}}$ (b) $\dfrac{1}{2\sqrt{a-x}\sqrt{x-b}}$

(c) $\dfrac{-1}{2\sqrt{x-a}\sqrt{x-b}}$ (d) $-\dfrac{1}{2\sqrt{a-x}\sqrt{x-b}}$

4. Let $f(x)=\begin{vmatrix} x^3 & \sin x & \cos \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$, where p is a constant.

Then $\dfrac{d^3}{dx^3}[(f(x)]$ at $x = 0$ is

(a) p (b) $p + p^2$
(c) $p + p^3$ (d) independent of p

5. If $\sin y = x\sin(a+y)$ and $\dfrac{dy}{dx}=\dfrac{A}{1+x^2-2x\cos a}$, then the value of A is

(a) $\sin a$ (b) $\cos a$
(c) $-\cos a$ (d) $-\sin a$

6. If $y=\sqrt{(x-1)(x-2)(x-3)(x-4)}$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{1}{2}y\left[\dfrac{1}{x-1}+\dfrac{1}{x-2}+\dfrac{1}{x-3}+\dfrac{1}{x-4}\right]$

(b) $\dfrac{y}{2}\left[\dfrac{1}{x-1}-\dfrac{1}{x-2}-\dfrac{1}{x-3}-\dfrac{1}{x-4}\right]$

(c) $y\left[\dfrac{1}{x-1}+\dfrac{1}{x-2}+\dfrac{1}{x-3}+\dfrac{1}{x-4}\right]$

(d) $\dfrac{y}{4}\left[\dfrac{1}{x-1}+\dfrac{1}{x-2}+\dfrac{1}{x-3}+\dfrac{1}{x-4}\right]$

7. If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\cdots\infty}}}}$, then $\dfrac{dy}{dx}$ is

(a) $\dfrac{(y^2-x)}{2y^3-2xy-1}$ (b) $\dfrac{y^2-x}{2y^3-2xy+1}$

(c) $\dfrac{y^2-x}{2y^3-xy-1}$ (d) $\dfrac{y^2-x}{2x^3-xy-1}$

8. If $f(x)=\cos^{-1}\left[\dfrac{1-(\log_e x)^2}{1+(\log_e x)^2}\right]$, then $f'(e)$ is equal to

(a) $\dfrac{2}{e}$ (b) $\dfrac{1}{e}$

(c) $\dfrac{3}{e}$ (d) $\dfrac{4}{e}$

9. If $y=x^{(\log x)^{\log^{\log x}}}$, then $\dfrac{dy}{dx}$ is equal to

(a) $\dfrac{y\log y}{x\log x}(2\log\log x+1)$

(b) $\dfrac{x\log x}{y\log y}(2\log\log x+1)$

(c) $\dfrac{2y\log y}{x\log x}(\log\log x+1)$

(d) none of these

10. If $f(9) = 9$ and $f'(9) = 4$, then $\lim\limits_{x\to 9}\dfrac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ equals

(a) 9 (b) 4
(c) 36 (d) none of these

11. If g is the inverse of f and $f'(x)=\dfrac{1}{1+x^{n'}}$, then $g'(x)$ equals

(a) $1 + x^n$ (b) $1 + [f(x)]^n$
(c) $1 + [g(x)]^n$ (d) none of these

12. $\dfrac{d}{dx}\cos^{-1}\left(\dfrac{4x^3}{27}-x\right)=$

(a) $\dfrac{3}{\sqrt{9-x^2}}$ (b) $\dfrac{-1}{\sqrt{9-x^2}}$

(c) $\dfrac{-3}{\sqrt{9-x^2}}$ (d) $\dfrac{1}{\sqrt{9-x^2}}$

13. If $ax^2+2hxy=by^2=1$, then $\dfrac{d^2y}{dx^2}$ is equal to

(a) $\dfrac{ab-h^2}{(hx+by)^2}$ (b) $\dfrac{h^2-ab}{(hx+by)^3}$

(c) $\dfrac{h^2+ab}{(hx+by)^3}$ (d) none of these

14. Let $f(t)$ – $\ln(t)$. Then $\dfrac{d}{dx}\left[\int_{x^2}^{3} f(t)dt\right]$

(a) has a value 0 when x = 0
(b) has a value 0 when x = 1 and x = 4/9
(c) has a value $9e^2 - 4e$ when $x = e$
(d) has a differential coefficient $27e - 8$ for $x = e$

15. If $y=\cos^{-1}\left(\dfrac{2\cos x-3\sin x}{\sqrt{13}}\right)$, then $\dfrac{dy}{dx}$ is

(a) zero (b) constant = 1
(c) constant ≠ 1 (d) none of these

16. If $\sqrt{\dfrac{v}{\mu}}+\sqrt{\dfrac{\mu}{v}}=6$ then $\dfrac{dv}{d\mu}=$

(a) $\dfrac{17\mu-v}{\mu-17v}$ (b) $\dfrac{\mu-17v}{17\mu-v}$

(c) $\dfrac{17\mu+v}{\mu-17v}$ (d) $\dfrac{\mu+17v}{17\mu-v}$

17. If $y=\cot^{-1}\sqrt{x^2-1}+\sec^{-1}x$, then $x\dfrac{dy}{dx}$ equals

(a) x (b) 1
(c) 0 (d) –1

18. If $f(x)=\left(\dfrac{a+x}{b+x}\right)^{a+b+2x}$, then $f'(0)$ is

(a) $\left(2\log\dfrac{a}{b}+\dfrac{a^2-b^2}{ab}\right)\left(\dfrac{a}{b}\right)^{a+b}$

(b) $\left(2\log\dfrac{a}{b}+\dfrac{b^2-a^2}{ab}\right)\left(\dfrac{a}{b}\right)^{a+b}$

(c) $\left(2\log\dfrac{a}{b}+\dfrac{a^2+b^2}{ab}\right)\left(\dfrac{a}{b}\right)^{a+b}$

(d) none of these

19. If $x^p\cdot y^q=(x+y)^{p+q}$, then $\dfrac{dy}{dx}$ is

(a) independent of p
(b) independent of q
(c) dependent on both p and q
(d) $\dfrac{y}{x}$

20. Let g be the inverse function of f and $f'(x)=\dfrac{x^{10}}{(1+x^2)}$.
If $g(2) = a$, then $g'(2)$ is equal to

(a) $\dfrac{a}{2^{10}}$ (b) $\dfrac{1+a^2}{a^{10}}$

(c) $\dfrac{a^{10}}{1+a^2}$ (d) $\dfrac{1+a^{10}}{a^2}$

ANSWERS

1. (a) **2.** (a) **3.** (a) **4.** (d) **5.** (a) **6.** (a) **7.** (a) **8.** (b) **9.** (a) **10.** (b)
11. (c) **12.** (b) **13.** (b) **14.** (b) (c) and (d) **15.** (b) **16.** (b) **17.** (c) **18.** (b)
19. (a) (b) and (d) **20.** (b)

Topic iii) Applications of Derivatives

1. The dimensions of the rectangle of maximum area that can be inscribed in the ellipse $(x/4)^2 + (y/3)^2 = 1$ are
(a) $\sqrt{8}, \sqrt{2}$ (b) 4, 3
(c) $2\sqrt{8}, 3\sqrt{2}$ (d) $\sqrt{2}, \sqrt{6}$

2. The maximum value of $x^{1/x}$ is
(a) $1/e$ (b) $e^{1/e}$
(c) e (d) $1/e^e$

3. If the curves $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ intersect orthogonally, then
(a) $\frac{1}{a}+\frac{1}{A}=\frac{1}{b}+\frac{1}{B}$ (b) $\frac{1}{a}-\frac{1}{A}=\frac{1}{b}-\frac{1}{B}$
(c) $\frac{1}{a}+\frac{1}{b}=\frac{1}{B}-\frac{1}{A}$ (d) $\frac{1}{a}-\frac{1}{b}=\frac{1}{A}-\frac{1}{B}$

4. Tangents are drawn to $x^2 + y^2 = 16$ from the point $P(0, h)$. These tangents meet the x-axis at A and B. If the area of the triangle PAB is minimum, then
(a) $h=12\sqrt{2}$ (b) $h=6\sqrt{2}$
(c) $h=8\sqrt{2}$ (d) $h=4\sqrt{2}$

5. A population $p(t)$ of 1000 bacteria introduced into the nutrient medium grows according to the relation $p(t)=1000+\frac{1000t}{100+t^2}$. The maximum size of this bacterial population is
(a) 1100 (b) 1250
(c) 1050 (d) 5250

6. In a regular triangular prism, the distance from the centre of one base to one of the vertices of the other base is l. The altitude of the prism for which the volume is greatest is
(a) $l/2$ (b) $l/\sqrt{3}$
(c) $l/3$ (d) $l/4$

7. The angle at which the curve $y = ke^{kx}$ intersects the y-axis is
(a) $\tan^{-1}(k^2)$ (b) $\cot^{-1}(k^2)$
(c) $\sin^{-1}\left(\frac{1}{\sqrt{1+k^4}}\right)$ (d) $\frac{\pi}{2}-\tan^{-1}(k^2)$

8. Two vertices of a rectangle are on the positive x-axis. The other two vertices lie on the circle $x^2+y^2=1$. Then the maximum area of the rectangle is
(a) 4/3 (b) 3/5
(c) 4/5 (d) 3/4

9. The lower corner of a leaf in a book is folded over so as to just reach the inner edge of the page. The fraction of width folded over if the area of the folded part is minimum is
(a) 5/8 (b) 2/3
(c) 3/4 (d) 4/5

10. The maximum value of $\frac{\log x}{x}$ is equal to
(a) $2/e$ (b) $1/e$
(c) e (d) 1

11. The minimum value of $\left(1+\frac{1}{\sin^n\alpha}\right)\left(1+\frac{1}{\cos^n\alpha}\right)$ is
(a) 1 (b) 2
(c) $(1+2^{n/2})^2$ (d) none of these

12. The length of the subtangent to the curve $x^2+xy+y^2=7$ at $(1, -3)$ is
(a) 3 (b) 5
(c) 3/5 (d) 15

13. The point(s) on the curve $y^3+3x^2=12y$ where the tangent is vertical is (are)
(a) $(\pm 4/\sqrt{3}-2)$ (b) $(\pm\sqrt{11/3},1)$
(c) (0, 0) (d) $(\pm 4/\sqrt{3},2)$

14. If PQ and PR are the two sides of a triangle, then the angle between them which gives maximum area of the triangle is
(a) π (b) $\pi/3$
(c) $\pi/4$ (d) $\pi/2$

15. The points of contact of the vertical tangents to $x = 2 - 3\sin\theta$ and $y = 3 + 2\cos\theta$ are
(a) (2, 5), (2, 1) (b) (–1, 3), (5, 3)
(c) (2, 5) (5, 3) (d) (–1, 3), (2, 1)

16. Let $f: R \to R$, where $f(x)=x^3-ax, a\in\mathbb{R}$. Then the set of values of a so that $f(x)$ is increasing in its entire domain is
(a) $(-\infty, 0)$ (b) $(0, \infty)$
(c) $(-\infty, \infty)$ (d) none of these

17. Let $f(x)=x-\sin x$ and $g(x)=x-\tan x$, where $x \in (0, \pi/2)$. Then for these values of x,
(a) $f(x)\,g(x)>0$ (b) $f(x)\,g(x)>0$
(c) $\frac{f(x)}{g(x)}>0$ (d) none of these

18. The function $f(x)=\sqrt{3}\sin x-\cos x$ will increase monotonically in the interval
(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{3},\frac{2\pi}{3}\right]$
(c) $[0, \pi]$ (d) none of these

19. Area of the triangle formed by the positive x-axis and the normal and the tangent to $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

(a) $2\sqrt{3}$ sq. units (b) $\sqrt{3}$ sq. units

(c) $4\sqrt{3}$ sq. units (d) none of these

20. The greatest value of the function

$f(x) = \dfrac{\sin 2x}{\sin[2x + (\pi/4)]}$ in the interval $[0, \pi/4]$ is

(a) $1/\sqrt{2}$ (b) $\sqrt{2}$

(c) 1 (d) $-\sqrt{2}$

ANSWERS

1. (c)	**2.** (b)	**3.** (b, d)	**4.** (d)	**5.** (c)	**6.** (b)	**7.** (b, d)	**8.** (c)	**9.** (b)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (d)	**15.** (b)	**16.** (a)	**17.** (b)	**18.** (b)	**19.** (a)	**20.** (b)