Book 6) Integral Calculus

Topics Covered: Indefinite Integration, Definite Integration & Its Applications.

Topic i) Indefinite Integration.

1. $I=\int \frac{(10x^9+10^x \log_e 10)}{(x^{10}+10^x)}dx$ is equal to

(a) $10^x+x^{10}+c$ (b) $10^x-x^{10}+c$
(c) 10^x+x^{10} (d) $\log_e(10^x+x^{10})+c$

2. $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx=$

(a) $e^x \cot x+c$ (b) $-e^x \cot x+c$
(c) $-e^x \tan x+c$ (d) $e^x \tan x+c$

3. $I=\int\sqrt{1+2\tan x(\sec x+\tan x)}dx$ is equal to

(a) $\log_e\left|\sec^2 x+\tan x \sec x\right|+c$
(b) $\log_e\left|1+\tan x(\sec x+\tan x\right|+c$
(c) $\log_e\left|\sin x(\sec x-\tan x)\right|+c$
(d) none of these

4. $\int\frac{x^2+x-1}{x^2+x-6}dx=$

(a) $x+\log(x+3)+\log(x-2)+c$
(b) $x-\log(x+3)+\log(x-2)+c$
(c) $x-\log(x+3)-\log(x-2)+c$
(d) none of these

5. If $A=\int e^x \sin x$ and $B=\int e^x \cos x dx$, then A^2+B^2 is equal to

(a) $\frac{e^{2x}}{2}$ (b) $\frac{1}{2}e$
(c) $\frac{e^{3x}}{3}$ (d) none of these

6. $\int\frac{e^x}{e^{2x}+5e^x+6}dx$ equals

(a) $\log\left(\frac{e^x+3}{e^x+2}\right)+c$ (b) $\log\left(\frac{e^x+2}{e^x+3}\right)+c$
(c) $\frac{1}{2}\log\left(\frac{e^x+2}{e^x+3}\right)+c$ (d) none of these

7. If $I=\int x\log_e\left(1+\frac{1}{x}\right)dx$

$=p(x)\log_e(x+1)+g(x)x^2+Lx+c$, then

(a) $p(x)=\frac{1}{2}x^2$ (b) $g(x)=\log_e x$
(c) $L=1$ (d) none of these

8. $\int \frac{dx}{4\sin^2 x + 4\sin x\cos x + 5\cos^2 x}$ is equal to

(a) $\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$ (b) $\frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(c) $4\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$ (d) none of these

9. If n is an odd positive integer, then $\int |x^n|\, dx$ is equal to

(a) $\left|\frac{x^{n+1}}{n+1}\right| + c$ (b) $\frac{x^{n+1}}{n+1} + c$

(c) $\frac{|x^n| x}{n+1} + c$ (d) none of these

10. $\int \frac{1}{4 + 5\sin x} dx$ is equal to

(a) $\frac{1}{3}\log\left|\frac{\tan x/2 + 5}{3}\right| + c$ (b) $\frac{1}{3}\log\left|\frac{2\tan x/2 + 1}{\tan x/2 + 2}\right| + c$

(c) $\frac{1}{3}\log\left|\frac{2\tan x/2 + 1}{\tan x/2 - 2}\right| + c$

(d) $\frac{1}{\sqrt{3}}\log\left|\frac{2\tan x/2 - 1}{\tan x/2 + 1}\right| + c$

11. $\int [1 + 2\tan x(\tan x + \sec x)]^{1/2} dx$ is equal to

(a) $\log\sec x(\sec x - \tan x) + c$

(b) $\log \text{cosec}(\sec x + \tan x) + c$

(c) $\log\sec x(\sec x + \tan x + c)$

(d) $\log(\sec x + \tan x) + c$

12. If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = A\cos 4x + c$, then A is

(a) 1/8 (b) –(1/8) (c) 1/4 (d) –(1/4)

13. $\int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx =$

(a) $\log(e^x + x^e)$ (b) $e\log(e^x + x^e)$

(c) $\frac{1}{e}\log(e^x + x^e)$ (d) none of these

14. The value of $\int \frac{ax^2 - b}{x\sqrt{c^2x^2 - (ax^2 + b)^2}} dx$ is

(a) $\sin^{-1}\left[\frac{ax + (b/x)}{c}\right] + k$ (b) $\sin^{-1}\left[\frac{ax^2 + (b/x^2)}{c}\right] + k$

(c) $\cos^{-1}\left[\frac{ax + (b/x)}{c}\right] + k$ (d) $\cos^{-1}\left(\frac{ax^2 + (b/x^2)}{c}\right) + k$

15. The value of $\int e^x \frac{1 + nx^{n-1} - x^{2n}}{(1 - x^n)\sqrt{1 - x^{2n}}} dx$ is

(a) $\frac{e^x\sqrt{1 - x^n}}{1 - x^n} + c$ (b) $e^x\frac{\sqrt{1 + x^{2n}}}{1 - x^{2n}} + c$

(c) $\frac{e^x\sqrt{1 - x^{2n}}}{1 - x^{2n}} + c$ (d) $e^x\frac{\sqrt{1 - x^{2n}}}{1 - x^n} + c$

16. $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx =$

(a) $\frac{12}{13} + \frac{5}{13}\log(3\cos x + 2\sin x) + c$

(b) $\frac{12}{13} - \frac{5}{13}\log(3\cos x + 2\sin x) + c$

(c) $\frac{5}{13}x + \frac{12}{3}\log(3\cos x + 2\sin x) + c$

(d) none of these

17. $\int x^x(1 + \log x)dx =$

(a) $x^x\log x + c$ (b) $\frac{x^x}{\log x} + c$

(c) $x^x + c$ (d) $x^x(x + 1) + c$

18. $\int \frac{dx}{x\log x\log(\log x)} =$

(a) $\log(\log x)$ (b) $\log[\log(\log x)]$

(c) $[\log(\log x)]^2$ (d) none of these

19. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx$ is equal to

(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + c$

(c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (d) $\frac{1}{3}\left(\frac{x+2}{x+1}\right)^{1/4} + c$

20. $\int \frac{1}{x}\left(\log_{ex} e \cdot \log_{e^2x} e \cdot \log_{e^3x} e\right) dx$ is equal to

(a) $\frac{1}{2}\log(\log_e e^2x) - \log(\log_e e^3x) + \log(\log_e e^4x) + c$

(b) $\frac{1}{2}\log(\log_e x) - \log(\log_e x) + \log(\log_e x) + c$

(c) $\frac{1}{2}\log(\log_e ex) - \log(\log_e e^2x^2) + \log(\log_e e^3x^3) + c$

(d) $\frac{1}{2}\log(\log_e ex) - \log(\log_e e^2x) + \log(\log_e e^3x) + c$

ANSWERS

1. (d)	**2.** (d)	**3.** (d)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (d)	**8.** (b)	**9.** (a)	**10.** (b)
11. (c)	**12.** (a)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (b)	**17.** (c)	**18.** (b)	**19.** (a)	**20.** (d)

Topic (ii) Definite Integrals.

1. $\lim_{h\to 0} \frac{\int_e^{x+h} \ln^2 t\,dt - \int_a^x \ln^2 t\,dt}{h}$ equals

(a) 0 (b) $\ln^2 x$
(c) $2 \ln x/x$ (d) does not exist

2. $\lim_{n\to\infty} \left[\frac{1}{2n} + \frac{1}{2n+1} + \cdots + \frac{1}{6n}\right]$ is equal to

(a) ln 3 (b) ln 6
(c) ln 2 (d) 2 ln 2

3. The value of the function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2\ln t)dt$, where $f'(x)$ vanishes, is

(a) e^{-1} (b) 0
(c) $2e^{-1}$ (d) $1 + 2e^{-1}$

4. The value of the integral $\int_{e^{-1}}^{e^2} \left|\frac{\log_e x}{x}\right| dx$ is

(a) 3/2 (b) 5/2
(c) 3 (d) 5

5. $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r^3}{r^4+n^4}\right)$ equals

(a) $\log 2$ (b) $\frac{1}{2}\log 2$

(c) $\frac{1}{3}\log 2$ (d) $\frac{1}{4}\log 2$

6. If $\int_0^x f(t)dt = x+\int_x^1 t.f(t)dt$, then the value of $f(1)$ is

(a) 1/2 (b) 0

(c) 1 (d) −(1/2)

7. $\lim_{n\to\infty}\sum_{r=2n+1}^{3n}\frac{n}{r^2-n^2}$ is equal to

(a) $\log\sqrt{\frac{2}{3}}$ (b) $\log\sqrt{\frac{3}{2}}$

(c) $\log\frac{2}{3}$ (d) $\log\frac{3}{2}$

8. Let $f(x)=x-[x]$ for every real number x, where [.] denotes the greatest integer function. Then $\int_{-1}^{1} f(x)dx$ is

(a) 1 (b) 2

(c) 0 (d) 1/2

9. The value of $\lim_{n\to\infty}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)\cdots\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}$ is

(a) $\frac{e^{\pi/2}}{2e^2}$ (b) $2e^2e^{\pi/2}$

(c) $\frac{2}{e^2}e^{\pi/2}$ (d) none of these

10. If f is a continuous function, then $\frac{1}{k}\int_{ak}^{bk} f\left(\frac{x}{k}\right)dx$ is

(a) $\frac{1}{k}\int_a^b f(x)dx$ (b) $\int_a^b f(x)dx$

(c) $k\int_a^b f(x)dx$ (d) none of these

11. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2f(x)=0$ for all non-zero x, then $\int_{\sin\theta}^{\operatorname{cosec}\theta} f(x)dx$ equals

(a) $\sin\theta+\operatorname{cosec}\theta$ (b) $\sin^2\theta$

(c) $\operatorname{cosec}^2\theta$ (d) none of these

12. If $I_1=\int_0^{n\pi} f(|\cos x|)dx$ and $I_2=\int_0^{5\pi} f(|\cos x|)dx$, then $(n\in N)$

(a) $nI_1=5I_2$ (b) $I_1+I_2=n+5$

(c) $\frac{I_1}{n}=\frac{I_2}{5}$ (d) none of these

13. $\lim_{n\to\infty}\left[\sin\frac{\pi}{2n}\cdot\sin\frac{2\pi}{2n}\cdot\sin\frac{3\pi}{2n}\cdots\sin\frac{(n-1)\pi}{n}\right]^{1/n}$ is equal to

(a) 1/2 (b) 1/3

(c) 1/4 (d) none of these

14. The value of k if $\int_0^{\pi/3}\frac{\cos x}{3+4\sin x}dx=k\cdot\log\left(\frac{3+2\sqrt{3}}{3}\right)$ is equal to

(a) 1/2 (b) 1/3

(c) 1/4 (d) 1/8

15. If $[x]$ stands for the greatest integer function, the value of $\int_4^{10}\frac{[x^2]}{[x^2-28x+196]+[x^2]}dx$ is

(a) 0 (b) 1

(c) 3 (d) none of these

16. The value of $\int_{-\pi/4}^{\pi/4}\frac{dx}{\sec^2 x(1+\sin x)}$ is

(a) $\pi/4$ (b) π

(c) $\pi/2$ (d) 2π

17. If $f(a+b-x)=f(x)$, then $\int_a^b xf(x)dx$ is equal to

(a) $\frac{a+b}{2}\int_a^b f(b-x)dx$ (b) $\frac{a+b}{2}\int_a^b f(x)dx$

(c) $\frac{b-a}{2}\int_a^b f(x)dx$ (d) none of these

18. Let $f(x)=\min(\tan x,\cot x), 0\le x\le\frac{\pi}{2}$, then $\int_0^{\pi/2} f(x)dx$ is equal to

(a) ln 2 (b) $\ln\sqrt{2}$

(c) ln 3 (d) none of these

19. If $\int_0^{100} f(x)dx=a$, then $\sum_{r=1}^{100}\left[\int_0^1 f(r-1+x)dx\right]=$

(a) $100a$ (b) a

(c) 0 (d) $10a$

20. If $f(x)=\int_{x^2}^{x^2+1}e^{-t^2}dt$, then $f(x)$ increases in

(a) (2, 2) (b) no value of x

(c) (0, ∞) (d) (−∞, 0)

ANSWERS

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (a)	**9.** (c)	**10.** (b)
11. (d)	**12.** (c)	**13.** (c)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (d)

Topic iii) Applications of Definite Integrals

1. The area of the curve $y^2 = (7 - x)(5 + x)$ above the x-axis and between the ordinates $x = -5$ and $x = 1$ is
   (a) 9π (b) 18π
   (c) 15π (d) none of these

2. The area bounded by $x^2 + y^2 - 2x = 0$ and $y = \sin\frac{\pi x}{2}$ in the upper half of the circle is
   (a) $\pi/2 - 4/\pi$ (b) $\pi/4 - 2/\pi$
   (c) $\pi - 8/\pi$ (d) none of these

3. The area common to the circle $x^2 + y^2 = 64$ and the parabola $y^2 = 12x$ is equal to

(a) $\frac{16}{3}(4\pi + \sqrt{3})$ (b) $\frac{16}{3}(8\pi - \sqrt{3})$

(c) $\frac{16}{3}(4\pi - \sqrt{3})$ (d) none of these

4. The area of the region bounded by $y = x^2$ and $y = \frac{2}{1+x^2}$ is

(a) $\pi - \frac{2}{3}$ (b) $\pi - \frac{1}{2}$

(c) $2\pi - \frac{2}{3}$ (d) none of these

5. The area of the region formed by

$x^2 + y^2 - 6x - 4y + 12 \le 0,\ y \le x$ and $x \le \frac{5}{2}$ is

(a) $\frac{\pi}{6} - \frac{\sqrt{3}+1}{8}$ (b) $\frac{\pi}{6} + \frac{\sqrt{3}+1}{8}$

(c) $\frac{\pi}{6} - \frac{\sqrt{3}-1}{8}$ (d) none of these

6. The curve $y = a\sqrt{x} + bx$ passes through the point (1, 2) and the area enclosed by the curve, the x-axis and the line $x = 4$ is 8 sq. units. The values of a and b are

(a) $a = 4, b = -2$ (b) $a = 3, b\ -1$

(c) $a = -3, b = 5$ (d) $a = -1, b = 3$

7. The area of the closed figure bounded by $y = x, y = -x$ and the tangent to the curve $y = \sqrt{x^2 - 5}$ at the point (3, 2) is

(a) 5 (b) $\frac{15}{2}$

(c) 10 (d) $\frac{35}{2}$

8. Let $f(x) = \min\left[\tan x, \cot x, \frac{1}{\sqrt{3}}\right], \forall x \in \left(0, \frac{\pi}{2}\right)$. Then the area bounded by $y = f(x)$ and the x-axis is

(a) $\log\left(\frac{4}{3}\right) + \frac{\pi}{6\sqrt{3}}$ (b) $\log\left(\frac{2}{\sqrt{3}}\right) + \frac{\pi}{12\sqrt{3}}$

(c) $\log\left(\frac{4}{3}\right) + \frac{\pi}{12\sqrt{3}}$ (d) none of these

9. If A_m represents the area bounded by the curve $y = \log x^m$, the x-axis and the lines $x = 1$ and $x = e$, then $A_m + mA_{m-1}$ is

(a) m (b) m^2

(c) $m^2/2$ (d) $m^2 - 1$

10. Let $f(x) = ax^2 + bx + c$, where $a \in \mathbb{R}^+$ and $b^2 - 4ac < 0$. The area bounded by $y = f(x)$, the x-axis and the lines $x = 0, x = 1$ is

(a) $\frac{1}{6}[3f(1) + f(-1) + 2f(0)]$

(b) $\frac{1}{12}[5f(1) + f(-1) + 8f(0)]$

(c) $\frac{1}{6}[3f(1) - f(-1) + 2f(0)]$

(d) $\frac{1}{12}[5f(1) - f(-1) + 8f(0)]$

11. Consider a rectangle $ABCD$ formed by the points $A \equiv (0,0), B \equiv (6,0), C \equiv (6,0)$ and $D \equiv (0,4)$. $P(x, y)$ is a moving interior point of the rectangle, moving in such a way that $d(P, AB) \le$ min. $\{(d(P, BC), d(P, CD), d(P, AD)\}$. Here, $d(P, AB)$, $d(P, BC)$, $d(P, CD)$ and $d(P, AD)$ represent the distances of the point P from the sides AB, BC, CD and AD, respectively. The area of the region representing all possible positions of the point P is

(a) 8 (b) 4

(c) 12 (d) 6

12. The area bounded by the curves $x = a\cos^3 t$ and $y = a\sin^3 t$ is

(a) $\frac{3\pi a^2}{8}$ (b) $\frac{3\pi a^2}{16}$

(c) $\frac{3\pi a^2}{32}$ (d) $3\pi a^2$

13. If the area bounded by the curve $y = f(x)$, the lines $x = 1$, $x = b$ and the x-axis is $(b-1)\cos(3b+4), b > 1$, then $f(x)$ is

(a) $(x-5)\sin(3x-4)$

(b) $(x-1)\sin(x+1) + (x+1)\cos(x-1)$

(c) $\cos(3x+4) - 3(x-1)\sin(3x+4)$

(d) $(x-5)\cos(3x+4)$

14. The area common to the region determined by $y \ge \sqrt{x}$ and $x^2 + y^2 < 2$ has the value

(a) $\pi - 2$ (b) $2\pi - 1$

(c) $3\pi - \sqrt{2}/3$ (d) none of these

15. The area bounded by the curve $y = \max.\{\sin x, \cos x\}$, the x-axis and between the lines $x = \frac{\pi}{4}$ and $x = 2\pi$ is

(a) $\frac{(4\sqrt{2}-1)}{\sqrt{2}}$ (b) $(4\sqrt{2}-1)$

(c) $\frac{(4\sqrt{2}-1)}{2}$ (d) none of these

16. Consider a triangle OAB formed by the points $O \equiv (0, 0)$ $A \equiv (2, 0)$, $B \equiv (1, \sqrt{3})$. $P(x, y)$ is an arbitrary interior point of the triangle, moving in such a way that $d(P, OA) + d(P, PB) + d(P, OB) = \sqrt{3}$, where $d(P, OA)$, $d(P, AB)$, $d(P, OB)$ represent the distances of 'P' from the sides OA, AB and OB, respectively. The area of the region representing all possible positions of the point P is

(a) $2\sqrt{3}$ (b) $\sqrt{6}$

(c) $\sqrt{3}$ (d) none of these

17. If A_1 is the area enclosed by the curve $xy = 1$, the x-axis and the ordinates $x = 1$, $x = 2$ and A_2 is the area enclosed by the curve $xy = 1$, the x-axis and the ordinates $x = 2$, $x = 4$, then

(a) $A_1 = 2A_2$ (b) $A_2 = 2A_1$

(c) $A_2 = 2A_1$ (d) $A_1 = A_2$

18. The slope to the curve $y = f(x)$ at $(x, f(x))$ is $2x + 1$. If the curve passes through the point (1, 2), then the area of the region bounded by the curve $y = f(x)$, the x-axis and the line $x = 1$ is

(a) $\frac{5}{6}$ (b) $\frac{6}{5}$

(c) $\frac{1}{6}$ (d) 6

ANSWERS

1. (a)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (b)	**10.** (d)
11. (a)	**12.** (a)	**13.** (c)	**14.** (d)	**15.** (a)	**16.** (c)	**17.** (d)	**18.** (a)		