Book 7) Matrices.

Topics Covered : Determinants & Matrices

Topic i) Determinants.

1. $\begin{vmatrix} \cos C & \tan A & 0 \\ \sin B & 0 & -\tan A \\ 0 & \sin B & \cos C \end{vmatrix}$ has the value

(a) 0 (b) 1
(c) $\sin A \sin B \sin C$ (d) none of these

2. The value of $\begin{vmatrix} x & x^2 - yz & 1 \\ y & y^2 - zx & 1 \\ z & z^2 - xy & 1 \end{vmatrix}$ is

(a) 1 (b) -1
(c) 0 (d) $-xyz$

3. If $\begin{vmatrix} x^k & x^{k+2} & x^{k+3} \\ y^k & y^{k+2} & y^{k+3} \\ z^k & z^{k+2} & z^{k+3} \end{vmatrix} = (x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$,

then

(a) $k=-3$ (b) $k=-1$
(c) $k=1$ (d) $k=3$

4. If $\sqrt{-1}=i$ and ω is a non-real cube root of unity, then the value of $\begin{vmatrix} 1 & \omega^2 & 1+i+\omega^2 \\ -i & -1 & -1-i+\omega \\ 1-i & \omega^2-1 & -1 \end{vmatrix}$ is equal to

(a) 1 (b) i
(c) ω (d) 0

5. If the determinant $\begin{vmatrix} a+p & l+x & u+f \\ b+q & m+y & v+g \\ c+r & n+z & w+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K is

(a) 6 (b) 7
(c) 8 (d) 9

6. The value of $\begin{vmatrix} i^m & i^{m+1} & i^{m+2} \\ i^{m+5} & i^{m+4} & i^{m+3} \\ i^{m+6} & i^{m+7} & i^{m+8} \end{vmatrix}$, where $i=\sqrt{-1}$, is

(a) 1 if m is a multiple of 4
(d) 0 for all real m
(c) $-i$ if m is a multiple of 3
(d) none of these

7. Let $\begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-1 & \lambda+4 & 3\lambda \end{vmatrix} = p\lambda^4+q\lambda^3+r\lambda^2+s\lambda+t$ be an identity in λ, where $p, q, r\ s, t$, are independent of λ, then the value of t is

(a) 4 (b) 0
(c) 1 (d) none of these

8. If n is not a multiple of 3 and 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^{2n} & 1 & \omega^n \\ \omega^n & \omega^{2n} & 1 \end{vmatrix}$ is equal to

(a) 0 (b) ω
(c) ω^2 (d) 1

9. If $\Delta(x) = \begin{vmatrix} 1 & \cos x & 1-\cos x \\ 1+\sin x & \cos x & 1+\sin x - \cos x \\ \sin x & \sin x & 1 \end{vmatrix}$, then $\int_0^{\pi/2} \Delta(x)dx$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 0 (d) $-\frac{1}{2}$

10. If a, b, c are real numbers, and

$D = \begin{vmatrix} a & 1+2i & 3-5i \\ 1-2i & b & -7-3i \\ 3+5i & -7+3i & c \end{vmatrix}$, then D is

(a) purely real (b) purely imaginary
(c) non-real (d) an integer

11. If $i = \sqrt{-1}$, $\sqrt[4]{1} = \alpha, \beta, \gamma, \delta$ then $\begin{vmatrix} \alpha & \beta & \gamma & \delta \\ \beta & \gamma & \delta & \alpha \\ \gamma & \delta & \alpha & \beta \\ \delta & \alpha & \beta & \gamma \end{vmatrix}$ is equal to

(a) i (b) $-i$
(c) 1 (d) 0

12. Let $\Delta = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then

(a) $x + a$ is a factor of Δ
(b) $(x + a)^2$ is a factor of Δ
(c) $(x + a)^3$ is a factor of Δ
(d) $(x + a)^4$ is not a factor of Δ

13. The system of equations $ax + 4y + z = 0$, $bx + 3y + z = 0$ and $cx + 2y + z = 0$ has non-trivial solutions if a, b, c are in

(a) A.P. (b) G.P.
(c) H.P. (d) none of these

14. The system of equations

$$2x - y + z = 0$$
$$x - 2y + z = 0$$
$$\lambda x - y + 2z = 0$$

has an infinite number of non-trivial solutions for

(a) $\lambda = 1$ (b) $\lambda = 5$
(c) 5 (d) no real value of λ

ANSWERS

1. (a) **2.** (c) **3.** (b) **4.** (d) **5.** (c) **6.** (b) **7.** (b) **8.** (a) **9.** (d) **10.** (a)
11. (d) **12.** (a), (b) and (d) **13.** (a) **14.** (c)

Topic (ii) Matrices.

1. If $A = \text{dig}\,(2, -1, 3)$ and $B = \text{dig}\,(-1, 3, 2)$, then $A^2B =$
   (a) dig (5, 4, 11) (b) dig (– 4, 3, 18)
   (c) dig (3, 1, 8) (d) B

2. If $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$, then $B^T A^T$ is
   (a) a null matrix
   (b) an identity matrix
   (c) a scalar matrix, but not an identity matrix
   (d) such that $Tr(B^T A^T) = 4$

3. Which of the following relations is true for $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$?
   (a) $(A+B)^2 = A^2 + 2AB + B^2$
   (b) $(A-B)^2 = A^2 - 2AB + B^2$
   (c) $AB = BA$
   (d) none of these

4. The trace of a skew-symmetric matrix is always equal to
   (a) $\sum a_{ij}$ (b) $\sum a_{ii}$
   (c) zero (d) none of these

5. Which of the following is incorrect?
   (a) $A^2 - B^2 = (A+B)(A-B)$
   (b) $(A^T)^T = A$
   (c) $(AB)^n = A^n B^n$, where A and B commute
   (d) $(A-I)(I+A) = 0 \Leftrightarrow A^2 = I$

6. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then value of X^n is
   (a) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (b) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$
   (c) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (d) none of these

7. If the matrix AB is a zero matrix, then
   (a) $A = 0$ or $B = 0$
   (b) $A = 0$ and $B = 0$
   (c) it is not necessary that either $A = 0$ or $B = 0$
   (d) all the above statements are wrong

8. If A^T is the transpose of a square matrix A, then

(a) $|A| \neq |A^T|$

(b) $|A| = |A^T|$

(c) $|A| + |A^T| = 0$

(d) $|A| = |A^T|$ only when A is symmetric

9. If $AB = 0$ for the matrices $A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $\phi - \theta$ is

(a) an odd multiple of $\pi/2$
(b) an odd multiple of π
(c) an even multiple of $\pi/2$
(d) 0

10. If A is a skew-symmetric matrix and n is an even positive integer, then A^n is
(a) a symmetric matrix
(b) a skew-symmetric matrix
(c) a diagonal matrix
(d) none of these

11. If I_n is the identity matrix of order n, then $(I_n)^{-1}$
(a) does not exist (b) equals I_n
(c) equals O (d) none of these

12. If A and B are symmetric matrices, then ABA is a
(a) symmetric matrix (b) skew-symmetric matrix
(c) diagonal matrix (d) scalar matrix

13. If A is a non-singular matrix and A^T denotes the transpose of A, then

(a) $|A| \neq |A^T|$ (b) $|A \cdot A^T| \neq |A|^2$

(c) $|A^T \cdot A| \neq |A^T|^2$ (d) $|A| + |A^T| \neq 0$

14. Given $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$; $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. If $A - \lambda I$ is a singular matrix, then

(a) $\lambda \in \phi$ (b) $\lambda^2 - 3\lambda - 4 = 0$

(c) $\lambda^2 + 3\lambda + 4 = 0$ (d) $\lambda^2 - 3\lambda - 6 = 0$

15. If for a matrix A of order 2, $A^2 + I = 0$, where I is the identity matrix, then A equals

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

16. From the matrix equation $AB = AC$, we conclude that $B = C$ provided
(a) A is singular (b) A is non-singular
(c) A is symmetric (d) A is square

17. If A is a square matrix of order 3, then the true statement is (where I is the unit matrix)
(a) $\det(-A) = -\det A$
(b) $\det A = 0$
(c) $\det(A + I) = 1 + \det A$
(d) $\det 2A = 2\det A$

18. If A is a square matrix of order $n \times n$ and k is a scalar, then adj (kA) is equal to
(a) k adj A (b) k^n adj A
(c) k^{n-1} adj A (d) k^{n+1} adj A

19. If A and B are symmetric matrices of order $n\,(A \neq B)$, then
(a) $A + B$ is a skew-symmetric matrix
(b) $A + B$ is a symmetric matrix
(c) $A + B$ is a diagonal matrix
(d) $A + B$ is a zero matrix

20. If $F(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $G(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, then $[F(\alpha)G(\beta)]^{-1} =$

(a) $F(\alpha) - G(\beta)$ (b) $-F(\alpha) - G(\beta)$

(c) $[F(\alpha)]^{-1}[G(\beta)]^{-1}$ (d) $[G(\beta)]^{-1}[F(\alpha)]^{-1}$

21. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 - (a + b)x + k = 0$, then
(a) $k = bc$ (b) $k = ad$
(c) $k = a^2 + b^2 + c^2 + d^2$ (d) $k = ad - bc$

22. The singularity of matrix

$$\begin{bmatrix} 1 & a & a^2 \\ \cos(p-d)x & \cos px & \cos(p+d)x \\ \sin(p-d)x & \sin px & \sin(p+d)x \end{bmatrix}$$

depends on which of the following parameters?
(a) a (b) p
(c) x (d) none of these

23. If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, then $AB =$

(a) A^3 (b) B^2
(c) 0 (d) I

24. Which of the following is true for matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(a) $A + 4I$ is a symmetric matrix
(b) $A - B$ is a diagonal matrix for any value of α if $B = \begin{bmatrix} \alpha & -1 \\ 2 & 5 \end{bmatrix}$
(c) $A - 4I$ is a skew-symmetric matrix
(d) none of these

25. The value of a for which the system of equations

$$a^3x + (a+1)^3 y + (a+2)^3 z = 0$$
$$ax + (a+1)y + (a+2)z = 0$$
$$x + y + z = 0$$

has a non-zero solution is

(a) -1 (b) 0
(c) 1 (d) none of these

26. The value of k for which the set of equations

$$3x + ky - 2z = 0, \quad x + ky + 3z = 0$$

and $$2x + 3y - 4z = 0$$

has a non-trivial solution over the set of rationals is

(a) $\frac{33}{2}$ (b) $\frac{31}{2}$
(c) 16 (d) 15

27. If $AB = A$ and $BA = B$, then B^2 is equal to

(a) B (b) A
(c) 1 (d) 0

28. The values of x for which the matrix

$$\begin{bmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{bmatrix}$$

is non-singular are

(a) $R - \{0\}$ (b) $R - \{-a(a+b+c)\}$
(c) $R - \{0, -(a+b+c)\}$ (d) none of these

29. Which of the following is a nilpotent matrix?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
(c) $\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

30. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19A^{-1}$ is equal to

(a) A^T (b) $2A$
(c) $\frac{1}{2}A$ (d) A

ANSWERS

1. (b)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (c)	**8.** (b)	**9.** (a)	**10.** (a)
11. (b)	**12.** (a)	**13.** (d)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (a)	**18.** (c)	**19.** (b)	**20.** (d)
21. (d)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (a)	**26.** (a)	**27.** (a)	**28.** (c)	**29.** (c)	**30.** (d)

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