Let $\tan(\alpha), \tan(\beta), \tan(\gamma)$ be roots of $x^3 - ax^2 + 3bx - 1 = 0$.

Suppose $(\tan(\alpha), \cot(\alpha))$, $(\tan(\beta), \cot(\beta))$, $(\tan(\gamma), \cot(\gamma))$ are vertices of of triangle. Then the sum of the abscissa & the ordinate of the centroid of the triangle is

A) $a+b+1$

B) $a+b$

C) $b+2a$

D) $a+2b$

$$\text{abscissa} = \frac{\tan(\alpha) + \tan(\beta) + \tan(\gamma)}{3} = \frac{3a}{3} = a.$$

$$\text{ordinate} = \frac{\cot(\alpha) + \cot(\beta) + \cot(\gamma)}{3} = \frac{1}{3} \times \frac{1}{\tan(\alpha)\tan(\beta)\tan(\gamma)} \times \left[\tan(\beta)\tan(\gamma) + \tan(\alpha)\tan(\gamma) + \tan(\alpha)\tan(\beta)\right]$$

$$= \frac{1}{3} \times \frac{1}{1} \times 3b = b$$

$\therefore$ addition $= a + b$.

Consider points on the parabola $y^2 = 4ax$, whose abscissae are in the ratio $k:1$. Tangents are drawn at these points. The locus of their point of intersection is a/an

A) Circle

B) Ellipse

C) Pair of straight lines.

D) Parabola.

Let the parameter values be t_1 & t_2. $\therefore \dfrac{at_1^2}{at_2^2} = k \Rightarrow t_1 = \sqrt{k}\, t_2$.

On solving the equations of tangents at (t_1) & (t_2),

$$x = at_1t_2 \quad \& \quad y = a(t_1 + t_2)$$

Refer any standard book for the result.

$$\therefore x = a \times \sqrt{k}\, t_2^2 \quad \& \quad y = a \times (\sqrt{k} + 1) t_2.$$

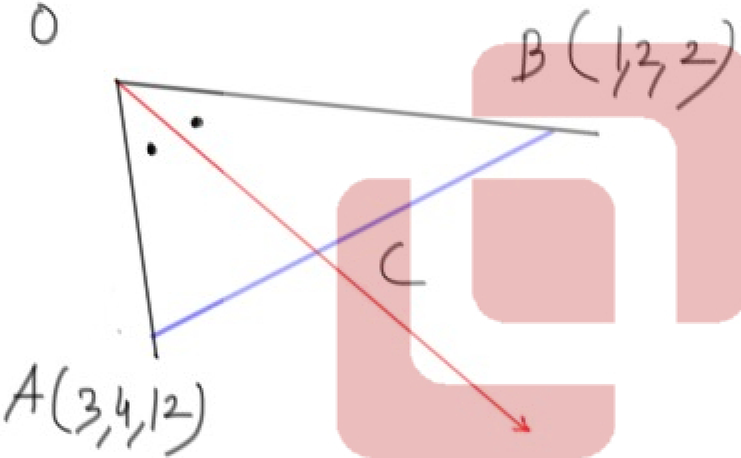
$$\therefore \frac{x}{a\sqrt{k}} = \frac{y^2}{a^2(\sqrt{k}+1)^2}$$

parabola. C

Let $A \equiv (3,4,12)$ & $B \equiv (1,2,2)$. The sum of x, y & z coordinates of the point on line AB, where bisector of $\angle AOB$ meets the line is

A) $\frac{51}{8}$ B) 6 C) 7 D) $\frac{61}{8}$.

(D)



$$\frac{AC}{CB} = \frac{OA}{OB} = \frac{13}{3}.$$

$\therefore$ C divides AB internally in 13:3.

$$\therefore C \equiv \left(\frac{13+9}{16}, \frac{26+12}{16}, \frac{26+36}{16} \right)$$

Addition of coordinates $= \frac{122}{16} = \frac{61}{8}.$

Let $\lim_{x \to 0^+} f(x) = A$ & $\lim_{x \to 0^-} f(x) = B$. Then

A) $\lim_{x \to 0^+} f(x^3 - x) = A$

C) $\lim_{x \to 0^+} f(x^2 - x^4) = B$

B) $\lim_{x \to 0^-} f(x^3 - x) = A$

D) $\lim_{x \to 0^-} f(x^2 - x^4) = B$.

$x>0$ & $x\to 0$ implies $0<x^3<x$ $\therefore$ $x^3-x=y$ is $-$ve.

$$\therefore \lim_{x\to 0^+} f(x^3-x) = \lim_{y\to 0^-} f(y) = B.$$

$x>0$ & $x\to 0$ implies $0<x^4<x^2$. $\therefore$ $x^2-x^4=z$ is +ve

$$\therefore \lim_{x\to 0^+} f(x^2-x^4) = \lim_{z\to 0^+} f(z) = A$$

$x<0$ & $x\to 0$ implies $0<x<x^3<0$ $\therefore$ $x^3-x=w$ is +ve.

$$\therefore \lim_{x\to 0^-} f(x^3-x) = \lim_{w\to 0^+} f(w) = A \quad \therefore \text{(B)}$$

Number of zeros of the function $x^3 - 15x + 1$ in $[-4, 4]$ is

A) 0 B) 1 C) 2 D) 3.

$f(-4) = -3,$ $\quad f(-1) = 15,$ $\quad f(1) = -13$ $\quad$ & $\quad f(4) = 5$

$\downarrow$ $-ve$ $\quad \downarrow$ $+ve$ $\quad \downarrow$ $-ve$ $\quad \downarrow$ $+ve$

1 $\quad$ 1 $\quad$ 1

$\therefore$ all zeros lie in $[-4, 4]$, by intermediate value theorem.

Let $f(x)$ satisfy the following $\forall\, x \,\&\, y \in \mathbb{R}$.

i) $f(x+y) = f(x) \times f(y)$.

ii) $f(x) = 1 + x \times g(x)$, where $\lim_{x \to 0} g(x) = 1$.

Then,

A) $f(x)$ is continuous everywhere, but is not differentiable at atleast 1 value.
B) $f(x)$ possesses at least 1 discontinuity, but is removable.
C) $f(x)$ is differentiable everywhere.
D) $f(x)$ is differentiable everywhere & $f'(x) = f(x)\ \forall\, x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(x) \times f(h) - f(x)}{h}$$

$$= f(x) \times \lim_{h \to 0} \frac{f(h) - 1}{h}$$

$$= f(x) \times \lim_{h \to 0} g(h)$$

$$= f(x) \times 1. \quad \therefore \text{(c)}$$

Let a & b be 2 positive real numbers with a<b. Then

$$\lim_{t \to 0} \left\{ \int_0^1 \left[bx + a(1-x) \right]^t dx \right\}^{1/t} \text{ is}$$

A) $e \left(\frac{b^b}{a^a} \right)^{1/b-a}$

B) $\frac{1}{e} \left(\frac{b^b}{a^a} \right)^{1/b-a}$

C) $e \left(\frac{a^a}{b^b} \right)^{1/b-a}$

D) $\frac{1}{e} \left(\frac{a^a}{b^b} \right)^{1/b-a}$

$$\int_0^1 \left(bx + ax(1-x)\right)^t dx = \int_a^b \frac{u^t}{(b-a)} du, \quad \text{by} \quad u = bx + a(1-x).$$

$$= \frac{b^{t+1} - a^{t+1}}{(t+1)(b-a)}.$$

Let $z = \lim_{t \to 0} \left[\frac{b^{t+1} - a^{t+1}}{(t+1)(b-a)} \right]^{1/t}$ $\therefore \ln(y) = \lim_{t \to 0} \frac{1}{t} \times \ln\left(\frac{b^{t+1} - a^{t+1}}{(t+1)(b-a)}\right) \Big\} \frac{0}{0}$

Applying l'Hopital's rule, $\ln(y) = \ln \frac{b^{b/b-a}}{e\, a^{a/b-a}}$

$$\therefore y = \frac{1}{e} \times \left(\frac{b^b}{a^a}\right)^{1/b-a}$$

Let $f(x)$ be defined on $[0, 2\pi]$ by

$$f(x) = \int_0^{\pi} \cos(t)\cos(x-t)\,dt.$$

Then $f_{min} =$

A) 0 B) $-\frac{\pi}{4}$ C) $-\pi$ D) $-\frac{\pi}{2}$.

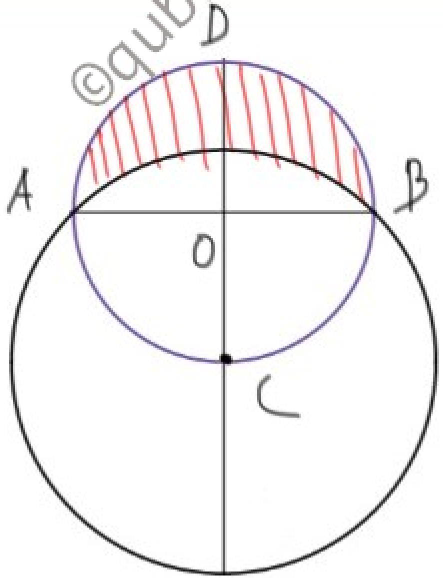
Using $\cos(A+B)+\cos(A-B)=2\cos(A)\cos(B)$, we get

$$f(x)=\frac{1}{2}\int_0^{\pi}\cos(x)+\cos(2t-x)\,dx=\frac{\pi\cos(x)}{2}+\left[\frac{\sin(2t-x)}{4}\right]_0^{\pi}=\frac{\pi\cos(x)}{2}$$

$\therefore f_{min}=-\frac{\pi}{2}$, at $x=\pi$

In the figure below, AB & CD are 2 perpendicular diameters with lengths 2 units each. With C as its center, a circular arc is drawn through points A & B. The shaded area is (in square units).

A) $\frac{\pi}{3}$ B) $\frac{2\pi}{5}$ C) $\frac{\pi}{2} - \frac{2}{3}$ D) 1



Radius of smaller circle $= 1$ unit & that of larger one is $\sqrt{2}$ units.

Required area $= \; 2 \times \left[\frac{\pi}{4} - \text{area of portion of larger circle in 1st quadrant} \right]$

Equation of larger circle : $(x)^2 + (y+1)^2 = 2$.

$$\therefore (y+1)^2 = 2 - x^2$$

$$\therefore y = \sqrt{2-x^2} - 1$$

you need to find this !

$$\therefore \text{required area} = \frac{\pi}{2} - 2 \times \left[\int_0^1 \sqrt{2-x^2} \right] - 1 \, dx = \frac{\pi}{2} - 2 \times \left(\frac{\pi}{4} + \frac{1}{2} \right) + 2$$

$$= 1.$$

The matrix equation $\begin{bmatrix} a & 1+a \\ 1-a & -a \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is true for

A) finitely many values of a.
B) countably infinite Values of a.
C) uncountably infinite Values of a, but not for all real numbers.
D) any real (a).

On squaring the LHS, we get

$$\begin{bmatrix} a & 1+a \\ 1-a & -a \end{bmatrix}\begin{bmatrix} a & 1+a \\ 1-a & -a \end{bmatrix} = \begin{bmatrix} a^2+1-a^2 & a+a^2-a-a^2 \\ a-a^2-a+a^2 & 1-a^2+a^2 \end{bmatrix}$$

Clearly, the square is independent of a. (D)

★ ie. you've got a way of generating as many (square roots) of $I_{2\times 2}$ as you want!

It is known that the number of items produced in a factory during a day is a random variable with mean 50. The probability that the daily production exceeds 75, i.e. $P(X>75)$

A) $\leq \frac{1}{2}$

B) $\leq \frac{2}{3}$

C) $\leq \frac{1}{3}$

D) cannot be estimated, as the distribution isn't completely known.

Suppose X is a random variable, taking non-negative values (e.g. items produced).

Further, let X have a continuous pdf $= f(x)$

Now, $E(X) = \int_0^{\infty} x \times f(x)\, dx$

For $a > 0$, $E(X) = \int_0^{a} x f(x)\, dx + \int_a^{\infty} x f(x)\, dx \geq \int_a^{\infty} x f(x)\, dx \geq \int_a^{\infty} a f(x)\, dx.$

$\therefore E(X) \geq a \times P(X \geq a)$ } This result is also known as Markov's inequality.

$\therefore 50 \geq 75 \times P(X \geq 75)$

$\therefore P(X \geq 75) \leq \frac{50}{75} = \frac{2}{3}$ (B)

Let there be a multiple-choice test with each question having only 1 out of (m) choices correct. It is mandatory to attempt all the questions. Ram is attempting the test with (p) being the probability that he knows the correct answer to a question. The probability that Ram knew the answer given that he answered correctly is

A) $\dfrac{mp}{1+p}$

B) $\dfrac{mp}{1+(m-1)p}$

C) $\dfrac{(1-m)p}{1+p}$

D) $\dfrac{(m-1)p}{1+(m-1)p}$

$$P(\text{he knew} / \text{he answered correctly}) = \frac{p}{p + \frac{1}{m} \times (1-p)} = \frac{mp}{mp + 1 - p}$$

$$= \frac{mp}{1 + (m-1)p}$$

The sum of the series $\sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right)$ is

A) $\ln(1/2)$
B) $\ln(2/\pi)$
C) $\ln(6/\pi^2)$
D) $-\infty$

$$\ln\left(1-\frac{1}{n^2}\right) = \ln\left(\frac{n^2-1}{n^2}\right) = \ln(n+1) + \ln(n-1) - 2\ln(n)$$

$$= \ln\left(\frac{n-1}{n}\right) - \ln\left(\frac{n}{n+1}\right)$$

Let the sum of first $(k-1)$ terms be s_k.

$$\therefore s_k = \sum_{n=2}^{k} \ln\left(1-\frac{1}{n^2}\right) = \sum_{n=2}^{k} \ln\left(\frac{n-1}{n}\right) - \ln\left(\frac{n}{n+1}\right) \quad \forall k \geq 2.$$

$$\therefore s_k = \left[\ln\left(\frac{1}{2}\right) - \ln\left(\frac{2}{3}\right)\right] + \left[\ln\left(\frac{2}{3}\right) - \ln\left(\frac{3}{4}\right)\right] + \cdots + \left[\ln\left(\frac{k-1}{k}\right) - \ln\left(\frac{k}{k+1}\right)\right]$$

$$= \ln\left(\frac{1}{2}\right) - \ln\left(\frac{k}{k+1}\right)$$

} such a series where intermediate terms get cancelled is known as a telescopic series!

$$\therefore \sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right) = \lim_{k\to\infty} \ln\left(\frac{1}{2}\right) - \ln\left(\frac{k}{k+1}\right)$$

$$= -\ln(2) \qquad \text{A}$$

How many times does the graph of $y = \dfrac{\sin(\pi \cos\theta)}{\cos(\pi \sin\theta)}$ meet the θ-axis in $[0, 2\pi]$?

A) 4 B) 5 C) 6 D) 7.

By letting $\theta \to \pi - \theta$ & $\theta \to \pi + \theta$, we establish that

$$f(\pi-\theta) = f(\pi+\theta) = \frac{-\sin(\pi\cos\theta)}{\cos(\pi\sin\theta)}$$ symmetry about $\theta = \pi$.

At $\theta = \pi$, $f(\pi) = \frac{0}{1} = 0$.

Thus, the graph meets the θ-axis an odd number of times.

$$f(0) = 0, \quad \lim_{\theta \to \frac{\pi}{6}^-} f(\theta) = \infty \ \& \ \lim_{\theta \to \frac{\pi}{6}^+} f(\theta) = -\infty, \quad f\left(\frac{\pi}{2}\right) = 0.$$

$\therefore$ we've $0, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}$ & π as the points where $f(\theta)$ meets the θ-axis.

Let $\cos^9(\theta) = \sum_{k=1}^{5} a_{2k-1} \cos\left[(2k-1)\theta\right]$. Then $a_5 =$

A) $9/256$

C) $9/64$

B) $21/64$

D) none of these

Let $\cos(\theta)+i\sin(\theta) = z$ & $\cos(\theta)-i\sin(\theta) = \frac{1}{z}$ $\left(= \frac{1}{e^{i\theta}} = e^{-i\theta}\right)$

$\therefore \quad z+\frac{1}{z} = 2\cos(\theta) \Rightarrow \left(z+\frac{1}{z}\right)^9 \times \frac{1}{2^9} = \cos^9(\theta)$

$${}^9C_1 = {}^9C_8 \qquad {}^9C_2 = {}^9C_7$$

$$\therefore \cos^9(\theta) = \frac{1}{2^9} \times \left[\left(z^9 + \frac{1}{z^9}\right) + 9\left(z^7 + \frac{1}{z^7}\right) + 36\left(z^5+\frac{1}{z^5}\right) + 84\left(z^3+\frac{1}{z^3}\right) + 126\left(z+\frac{1}{z}\right)\right]$$

$${}^9C_3 = {}^9C_6 \qquad {}^9C_4 = {}^9C_5$$

$$= \frac{1}{2^9} \times \left[2\cos(9\theta) + 18\cos(7\theta) + 72\cos(5\theta) + 168\cos(3\theta) + 252\cos(\theta)\right]$$

For a_5, $2k-1=5$, coefficient of $\cos(5\theta)$ is $\frac{72}{2^9} = \frac{9}{64}$