Book on Miscellaneous Topics : Logic, Induction, Binomial Theorem & sets and Relations.

Topic i) Logic

1. If p and q are true and r and s are false then which of the following expressions has the truth value false?
(a) $p \vee (q \vee r)$
(b) $[p \wedge (q \wedge r)] \vee [(p \vee q) \wedge (r \vee s)]$
(c) $[\sim (p \wedge q) \vee \sim r] \vee \{[(\sim p \vee q) \vee \sim r] \wedge s\}$
(d) none of three

2. Which of the following is not a tautology?
(a) $(p \wedge q) \Rightarrow p$
(b) $(p \wedge q) \Rightarrow (p \Rightarrow q)$
(c) $\sim (p \wedge q) \vee [(\sim p) \wedge q] \vee p$
(d) $(q \Rightarrow p) \Rightarrow p$

3. All equivalent formula for $p \wedge (q \Leftrightarrow r)$ which is free from biconditionals as wall as conditionals is
(a) $p \vee [(\sim q \vee r) \wedge (\sim r \vee q)]$
(b) $p \wedge [(\sim q \vee r) \wedge (\sim r \vee q)]$
(c) $p \wedge [(\sim q \wedge r) \wedge (\sim r \vee q)]$
(d) $p \wedge [(\sim q \vee r) \wedge (\sim r \wedge q)]$

4. Choose the correct alternative form the following:
(i) $(p \wedge q) \rightarrow (p \vee q)$ is a theorem
(ii) $(p \wedge q) \rightarrow (p \vee q)$ is a fallacy
(iii) $\sim (p \vee q) \leftrightarrow (\sim p \vee \sim q)$ is a fallacy
(iv) $(p \wedge q) \rightarrow (p \vee q)$ is a theorem
(a) (i) only (b) (i) and (iv)
(c) (i) and (iii) (d) none of these

5. The dual of the statement $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ is
(a) $(p \vee q) \wedge r \equiv p \wedge (q \vee r)$
(b) $(p \vee q) \vee r \equiv p \wedge (q \vee r)$
(c) $(p \vee q) \vee r \equiv p \vee (q \vee r)$
(d) $(p \wedge q) \wedge r \equiv p \wedge (q \vee r)$

6. If p: 2 is an even number,
q: 2 is a prime number, and $r : 2 + 2 = 2^2$,
then the symbolic statement $p \rightarrow (q \vee r)$ means
(a) 2 is an even number and 2 is a prime number or $2 + 2 = 2^2$
(b) 2 is an even number then 2 is a prime number or $2 + 2 = 2^2$
(c) 2 is an even number or 2 is a prime number then $2 + 2 = 2^2$
(d) If 2 is not an even number, then 2 is a prime number or $2 + 2 = 2^2$

7. Let p : I am brave,
q : I will climb the mount Everest
The symbolic form of a statement,
'I am neither brave nor I will climb the mount Everest' is
(a) $p \wedge q$ (b) $\sim (p \wedge q)$ (c) $\sim p \wedge \sim q$ (d) $\sim p \wedge q$

8. Which of the following is not logically equivalent to the proposition: 'A real number is either rational or irrational'.
(a) If a number is neither rational nor irrational, then it is not real
(b) If a number is not a rational or not an irrational, then it is not real
(c) If a number is not real, then it is neither rational nor irrational
(d) If a number is real, then it is rational or irrational

9. The negation of the proposition: "If we control population growth, we prosper" is
(a) If we do not control population growth, we prosper
(b) If we control population growth, we do not prosper
(c) We control population but we do not prosper
(d) We do not control population, but we prosper

10. If p is any statement, t is tautology and c is a contradiction, then which of the following is not correct?
(a) $p \vee (\sim p) = c$ (b) $p \vee t = t$
(c) $p \wedge t = p$ (d) $p \wedge c = c$

11. Which of the following is a contradiction?
(a) $(p \wedge q) \wedge \sim(p \vee q)$ (b) $p \vee (-p \wedge q)$
(c) $(p \Rightarrow q) \Rightarrow p$ (d) none of these

12. Which of the following is the inverse of the proposition: 'If a number is a prime, then it is odd'?
(a) If a number is not prime, then it is odd
(b) If a number is not prime, then it is even
(c) If a number is not odd, then it is not a prime
(d) If a number is not odd, then it is prime

13. The inverse of the statement 'If x is zero, then we cannot divide by x' is
(a) If we cannot divide by x, then x is zero
(b) If we cannot divide by x, then x is not zero
(c) If x is not zero, then we divide by x
(d) none of three

14. The negation of the statement $(p \vee q) \wedge r$ is
(a) $(\sim p \vee \sim q) \vee \sim q$ (b) $(\sim p \vee \sim q) \vee \sim r$
(c) $\sim(p \vee q) \to r$ (d) $p \wedge q$

ANSWERS

1. (d)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (b)	**9.** (c)	**10.** (a)
11. (a)	**12.** (b)	**13.** (c)	**14.** (b)						

Topic ii) Mathematical Induction

1. $3+3^2+3^3+\cdots+n$ terms is equal to

(a) $\frac{3}{2}(3^n+1)$ (b) $\frac{3}{2}(3^n-1)$

(c) $3(3^n-1)$ (d) $3(3^n+1)$

2. $1+4+13+40+\cdots+n$ terms is equal to

(a) $\frac{3^{n+1}-2^n}{2n}$ (b) $\frac{3^{n+1}-2n-3}{4}$

(c) $\frac{3n-1+3n^n}{9}$ (d) $\frac{3^{n+1}+2n^2}{8}$

3. $1+3+7+15+\cdots+n$ terms is equal to

(a) $2^{n+1}-n-2$ (b) n^2+n-2

(c) 2^n+n^2-n (d) none of these

4. Sum of the series $S=1^2-2^2+3^2-4^2+\cdots-2002^2+2003^2$ is equal to

(a) 2,007,006 (b) 1,005,004

(c) 2,000,506 (d) none of these

5. The positive interger n for which $2\times2^2+3\times2^3+4\times2^4+\cdots+n\times2^n=2^{n+10}$ is equal to

(a) 510 (b) 511

(c) 512 (d) 513

6. $1\underline{|1}+2\underline{|2}+3\underline{|3}+\cdots+n\underline{|n}$ is equal to

(a) $\underline{|n+1}-1$ (b) $\underline{|n-1}+1$

(c) $\underline{|n+1}+1$ (d) $\underline{|n-1}-1$

7. $2\cdot4+4\cdot7+6\cdot10+\cdots+(n-1)$ terms is equal to

(a) $2n^3-2n^2$ (b) $\frac{n^3+3n^2+1}{6}$

(c) $2n^3+2n$ (d) none of these

8. $3\cdot6+4\cdot7+5\cdot8+\cdots+(n-2)$ terms is equal to

(a) n^3+n^2+n+2 (b) $\frac{2n^3+12n^2+10n-84}{6}$

(c) $\alpha n^3+\beta n^2+rn$ (d) $\frac{n^3+6n^2+5n-42}{3}$

9. $1\cdot2\cdot3+2\cdot3\cdot4+3\cdot4\cdot5+n$ terms is equal to

(a) $\frac{n(n+1)(n+2)(3n+5)}{12}$ (b) $\frac{n(n+1)(n+2)(n+3)}{4}$

(c) $2n(n+1)(n+2)(n+3)$ (d) $\frac{n(n+1)(n+2)(n+3)}{12}$

10. $(1)+(1+3)+(1+3+5)+\cdots+n$ brackets is equal to

(a) $\frac{n(n+1)(n+2)}{6}$

(b) $\frac{n(n+1)(3n^2+23n+46)}{12}$

(c) $\frac{2(27n^3+90n^2+450-50)}{4}$

(d) $\frac{n(n+1)(2n+1)}{6}$

11. The sum of $(1^2)+(1^2+2^2)+(1^2+2^2+3^3)+\cdots+n$ brackets is equal to

(a) $\frac{n(n+1)(n+2)}{6}$

(b) $\frac{n(n+1)^2(n+2)}{12}$

(c) $\frac{2(27n^3+90n^2+450-50)}{4}$

(d) $\frac{n(n+1)(2n+1)}{6}$

12. The sum of n brackets of $(1)+\left(\frac{1}{3}+\frac{1}{3^2}\right)+\left(\frac{1}{3^3}+\frac{1}{3^4}+\frac{1}{3^5}\right)+\cdots$ is equal to

(a) $\frac{(3^n-1)^3}{2\cdot4^{n-1}}$ (b) $\frac{3^n-1}{2\cdot3^{(n-1)(n+2)/2}}$

(c) $\frac{3^n+1}{3\cdot7^{n-1}}$ (d) none of these

13. $(1)+(2+3+4)+(5+6+7+8+9)+\cdots+n$ brackets is equal to

(a) $(n-1)^3+n^3$ (b) $(n+1)^3+8n^2$

(c) $\dfrac{(n+1)(n+2)}{6n}$ (d) $(n+1)^3+n^3$

14. $(1)+(2+3)+(4+5+6)+\cdots+n$ brackets is equal to

(a) $\dfrac{n(n+1)(n^2+n+2)}{8}$ (b) $\dfrac{n(n+1)(n^2-n+2)}{8}$

(c) $\dfrac{n(n-1)(n^2+n+2)}{8}$ (d) $\dfrac{n(n-1)(n^2-n+2)}{8}$

15. The value of the sum in the nth bracket of $(1)+(2+3)+(4+5+6+7)+(8+9+10+\cdots+15)+\cdots$ is equal to

(a) $2^n(2^n+2^{n-1}-1)$ (b) $2^{n-1}(2^n+2^{n-1}-1)$

(c) $2^{n-2}(2^n+2^{n-1}-1)$ (d) none of these

16. If the natural numbers are divided into groups of $\{1\}$ $\{2, 3\}$ $\{4, 5, 6\}$ $\{7, 8, 9, 10\}$ ..., then the sum of 50 groups is equal to

(a) 62,525 (b) 65,225

(c) 56,255 (d) 55,625

17. $(1)+(2+3+4)+(5+6+7+6+9)+\cdots+n$ brackets is equal to

(a) $\dfrac{n(1+2)(n+2)}{6}$ (b) $\dfrac{n^2(n^2+1)}{2}$

(c) $n(n+1)(2n+1)$ (d) $\dfrac{n(n+1)(2n+1)}{6}$

ANSWERS

1. (b)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (d)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (b)	**10.** (d)
11. (b)	**12.** (b)	**13.** (a)	**14.** (a)	**15.** (c)	**16.** (a)	**17.** (b)			

Topic (iii) Binomial Theorem

1. The 14th term from the end in the expansion of $(\sqrt{x}-\sqrt{y})^{17}$ is

(a) ${}^{17}C_5 x^6(-\sqrt{y})^5$ (b) ${}^{17}C_6(\sqrt{x})^{11}y^3$

(c) ${}^{17}C_4 x^{13/2}y^2$ (d) none of these

2. If $n \in N$, then the sum of the coefficients in the expansion of the binomial $(5x-4y)^n$ is

(a) 1 (b) -1

(c) 1 (d) 0

3. When 5^{99} is divided by 13, the remainder is

(a) 8 (b) 9

(c) 10 (d) none of these

4. The number of values of r satisfying the equation ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is

(a) 1 (b) 2

(c) 3 (d) 4

5. The number of elements in the set of values of r for which ${}^{18}C_{r-2} + 2\cdot{}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ is satisfied is

(a) 4 (b) 5

(c) 7 (d) 10

6. If x is nearly equal to one, then the approximate value of $\frac{mx^m - nx^n}{m-n}$ is

(a) x^m (b) x^n

(c) x^{m+n} (d) none of these

7. If the coefficients of the pth, $(p+1)$th and $(p+2)$th terms in the expansion of $(1+x)^n$ are in A.P., then

(a) $n^2 - 2np + 4p^2 = 0$

(b) $n^2 - n(4p+1) + 4p^2 - 2 = 0$

(c) $n^2 - n(4p+1) + 4p^2 = 0$

(d) none of these

8. ${}^nC_0 - \frac{1}{2}{}^nC_1 + \frac{1}{3}{}^nC_2 - \cdots + (-1)^n \frac{{}^nC_n}{n+1} =$

(a) n (b) $\frac{1}{n}$

(c) $\frac{1}{n+1}$ (d) $\frac{1}{n-1}$

9. The expansion of $\frac{1}{(4-3x)^{1/2}}$ by binomial theorem will be valid, if

(a) $x<1$ (b) $|x|<1$

(c) $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$ (d) none of these

10. The two consecutive terms in the expansion of $(3+2x)^{74}$ with equal coefficients are

(a) 30th and 31st terms

(b) 29th and 30th terms

(c) 31st and 32nd terms

(d) 28th and 29th terms

11. If $\sum_{r=m}^{n} {}^{r}C_m = {}^{n+1}C_{m+1}$, then $\sum_{r=m}^{n}(n-r+1)\,{}^{r}C_m$ is equal to

(a) ${}^{n+3}C_{m+2}$ (b) ${}^{n+2}C_{m+2}$

(c) ${}^{n+2}C_{m+1}$ (d) none of these

12. In the expansion of $(7^{1/3}+11^{1/9})^{65/61}$, the number of terms free from radicals is

(a) 730 (b) 715

(c) 725 (d) 750

13. $(1+x+x^2+x^3)^5 = a_0+a_1x+a_2x^2+\cdots+a_{15}x^{15}$, then a_{10} is equal to

(a) 99 (b) 101

(c) 100 (d) 110

14. If $C_0, C_1, C_2, ..., C_n$ are the binomial coefficients in the expansion of $(1+x)^n$, n being even, then $C_0+(C_0+C_1)+(C_0+C_1+C_2)+\cdots+(C_0+C_1+C_2+\cdots+C_{n-1})$ is equal to

(a) $n\cdot 2^n$ (b) $n\cdot 2^{n-1}$

(c) $n\cdot 2^{n-2}$ (d) $n\cdot 2^{n-3}$

15. If $(1+x+2x^2)^{20} = a_0+a_1x+a_2x^2+\cdots+a_{40}x^{40}$, then $a_0+a_2+a_4+\cdots+a_{38}$ is equal to

(a) $2^{19}(2^{30}+1)$ (b) $2^{19}(2^{20}-1)$

(c) $2^{19}(2^{19}-1)$ (d) none of these

16. $C_1+2C_2+3C_3+4C_4+\cdots+nC_n =$

(a) 2^n (b) $n\cdot 2^n$

(c) $n\cdot 2^{n-1}$ (d) $n\cdot 2^{n+1}$

17. $\frac{1}{\sqrt[3]{6-3x}} =$

(a) $6^{\frac{1}{3}}\left[1-\frac{x}{6}-\frac{2x^2}{6^2}-\cdots\right]$

(b) $6^{-\frac{1}{3}}\left[1+\frac{x}{6}+\frac{2x^2}{6^2}+\cdots\right]$

(c) $6^{\frac{1}{3}}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-\cdots\right]$

(d) $6^{-\frac{1}{3}}\left[1-\frac{x}{6}+\frac{2x^2}{6^2}-\cdots\right]$

18. If the coefficients of 5th, 6th and 7th terms in the expansion of $(1+x)^n$ are in A.P., then $n =$

(a) 7 only (b) 14 only

(c) 7 or 14 (d) none of these

19. The integer next above to $(\sqrt{3}+1)^{2m}$ contains

(a) 2^{m+1} as a factor

(b) 2^{m+2} as a factor

(c) 2^{m+3} as a factor

(d) 2^m as a factor

20. If ${}^{13}C_r$ is denoted by C_r, then the value of $C_1+C_5+C_7+C_9+C_{11}$ is equal to

(a) $2^{12}-287$ (b) $2^{12}-165$

(c) $2^{12}-C_3$ (d) $2^{12}-C_2-C_{13}$

21. Let n be an odd natural number greater than 1. Then the number of zeros at the end of the sum $99n+1$ is

(a) 3 (b) 4

(c) 2 (d) none of these

22. The number of terms in the expansion of $\left(x^2+1+\frac{1}{x^2}\right)^n$, $n \in N$, is

(a) $2n$ (b) $3n$

(c) $2n+1$ (d) $3n+1$

23. In the expansion of $\left(x^3-\frac{1}{x^2}\right)^n$, $n\in N$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is

(a) 25 (b) 20

(c) 15 (d) none of these

24. If $(1+x)^n = \sum_{r=0}^{n} a_r x^r$, $b_r = 1+\frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^{n} b_r = \frac{(101)^{100}}{100!}$ then n is equal to

(a) 99 (b) 100

(c) 101 (d) none of these

25. If a and d are two complex numbers, then the sum to $(n + 1)$ terms of the following series

$aC_0 - (a+d)C_1 + (a+2d)C_2 - \cdots + \cdots$ is

(a) $\frac{a}{2^n}$ (b) na

(c) 0 (d) none of these

26. The digit at units place in the number $17^{1995} + 11^{1995} - 7^{1995}$ is

(a) 0 (b) 1

(c) 2 (d) 3

27. The interval in which x must lie so that the greatest term in the expansion of $(1 + x)^{2n}$ has the greatest coefficient is

(a) $\left(\frac{n-1}{n}, \frac{n}{n-1}\right)$ (b) $\left(\frac{n}{n+1}, \frac{n+1}{n}\right)$

(c) $\left(\frac{n}{n+2}, \frac{n+2}{n}\right)$ (d) none of these

28. The integral part of $(4+2\sqrt{5})^{2n+1}$ $(n \in N)$ is

(a) an even number

(b) an odd number

(c) an even or an odd number depending upon the value of n

(d) none of these

29. If the term independent of x in $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405, then the value of k is

(a) 2 (b) –2

(c) 3 (d) none of these

30. The coefficient of x^{11} in the expansion of $(2x^2 + x - 3)^6$ is

(a) 384 (b) 192

(c) 572 (d) 64

ANSWERS

1. (c)	**2.** (a)	**3.** (a)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (b)	**8.** (c)	**9.** (d)	**10.** (a)
11. (b)	**12.** (a)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (c)	**17.** (b)	**18.** (c)	**19.** (a)	**20.** (a)
21. (c)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (a)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (b)

Topic iv) Sets & Relations

1. In a group of 500 persons, if 350 can speak Hindi and 250 can speak English, then the number of persons speaking both the languages are
(a) 150 (b) 200
(c) 100 (d) 250

2. In a group of 150 persons, if 35 drink milk but not tea and 65 drink milk, then the number of persons who drink both tea and milk are
(a) 25 (b) 30
(c) 35 (d) none of these

3. Cardinal number of $P(P(P(\phi)))$ is
(a) 1 (b) 2
(c) 3 (d) 4

4. A class of 100 students sat in an examination in maths and physics. Every student cleared at least 1 subject, 55 students cleared maths and 67 students cleared physics. Then the number of students who cleared physics only is
(a) 22 (b) 33
(c) 10 (d) 45

5. The smallest set such that $A \cup \{1,2\} = \{1,2,3,5,9\}$ is
(a) $\{1,2\}$ (b) $\{1,2,3,5,9\}$
(c) $\{3,5,9\}$ (d) none of these

6. Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of the set of all determinants with value –1. Then:
(a) B has as many elements as C
(b) $C = \phi$
(c) B has twice as many elements as C
(d) $A = B \cup C$

7. Let a binary operation '$\forall$' on Q (set of all rational numbers) be defined by $a \circ b = a + 2b$ for all $a, b \in Q$, then
(a) Q is closed under the given operation
(b) the given operation is commutative
(c) the given operation is associative
(d) Q is not closed

8. Let S be the set of integers. For $a, b \in S$, aRb if and only if $|a-b|<1$, then
(a) R is not reflexive
(b) R is not symmetric
(c) $R = r = \{(a, a); a \in 1\}$
(d) R is not an equivalence relation

9. Let I be the set of integers, N the set of non-negative integers, N_p the set of non-positive integers, E the set of even integers and P the set of prime numbers. Then
(a) $N \cap N_p = \phi$ (b) $I \sim N = N_p$
(c) $N \Delta N_p = I \sim \{0\}$ (d) $E \cap P = \phi$

10. X and Y are two sets and X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements. Then the number of elements in Y is
(a) 30 (b) 10
(c) 40 (d) 50

11. R is the relation "x is a brother of y" in the set of children in a family. Then R is
(a) reflexive (b) symmetric
(c) transitive (d) none of these

12. Let $f: N \to N$ be defined as

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$$

Then f is
(a) one-to-one (b) one-to-one and onto
(c) onto (d) none of these

13. For the sets of real numbers given by

$R_1 = \{(x, y): x \in \mathbb{R}, y \in \mathbb{R}\ x^2 + y^2 \le 25\}$

$R_2 = \{(x, y): x \in \mathbb{R}, y \in \mathbb{R},\ 9y \ge 4x^2\}$

$R_1 \cap R_2$ is
(a) one-to-one
(b) onto
(c) one-to-one and onto
(d) none of these

14. A survey showed that 60% of the Indians like oranges whereas 75% like mangoes. If k % of the Indians like both oranges and mangoes, then
(a) $35 \le k \le 60$ (b) $45 \le k \le 60$
(c) $35 \ge k$ and $k \ge 60$ (d) none of these

15. If $A = \{3, 5\}$ and $B = \{5, 6, 7\}$, then $(A \cap B) \times (A \cup B)$ is
(a) $\{(5, 3)\ (5, 4)\ (5, 6)\}$
(b) $\{(5, 3), (5, 5), (5, 6), (5, 7)\}$
(c) $\{(6, 5), (6, 3), (6, 6), (6, 7)\}$
(d) none of these

16. If A and B are two sets and U is the universal set such that $n(U) = 800$, $n(A) = 300$, $n(B) = 400$ and $n(A \cap B) = 150$, then $n(A' \cap B')$ is equal to
(a) 400 (b) 150
(c) 350 (d) 250

17. If $A \subseteq B$, then
(a) $A' \subseteq B'$ (b) $A' = B'$
(c) $B' \subseteq A'$ (d) $B' \not\subseteq A'$

18. If $A = \{x, y, t\}$, $B = \{x, z, u\}$ and $C = \{x, z, v\}$, then
(a) $A - B = \{y, u\}$
(b) $B - C = \{z, v\}$
(c) $C - A = \{x, t\}$
(d) none of these

19. Let $A = Z \cup \{\sqrt{2}\}$. Define a relation R in A by aRb if and only if $a + b \in Z$. The relation R is then
(a) reflexive
(b) symmetric and transitive
(c) only transitive
(d) none of these

20. If $f(x) = 1 - \frac{1}{x}$, then $f\left[f\left(\frac{1}{x}\right)\right]$ is
(a) $\frac{1}{x}$ (b) $\frac{1}{1+x}$
(c) $\frac{x}{x-1}$ (d) $\frac{1}{x-1}$

ANSWERS

1. (c)	**2.** (b)	**3.** (d)	**4.** (d)	**5.** (c)	**6.** (c)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)
11. (c)	**12.** (c)	**13.** (d)	**14.** (a)	**15.** (b)	**16.** (d)	**17.** (c)	**18.** (d)	**19.** (b)	**20.** (d)